Multi-period Hub Location Problem with Serial Demands: A Case Study of Humanitarian Aids Distribution in Lebanon

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Abstract

In this paper, we address the problem of humanitarian aids distribution across refugee camps in war-ridden areas from a network design perspective. We show that the problem can be modeled as a variant of multi-period hub location problem with a particular demand pattern resulted by the user's behavior. The problem has been motivated by a case study of Lebanese experience in Syrian war refugee accommodation. We elaborate on the complexity and real-life constraints and, propose a compact formulation of a mathematical model of the problem. We then show that modeling the problem using a Benders paradigm drives $\mathcal{O}(n^3)$ variables of the original compact model unnecessary in addition to the constraints that are being projected out in a typical Benders decomposition. Additionally, we identify several classes of valid inequalities together with efficient separation procedures leading to a cut-and-Benders approach. Our extensive computational experiments on the case study with real data as well as randomly generated instances proves the performance of proposed solution methods.

Keywords: Hub-and-spoke network design, distribution, humanitarian aids, refugees, mateheuristics.

1. Introduction

The emergency situations including wars and natural disasters introduce fragility for civilians and occurrence of urgent requirements. Loosely speaking, in cases of natural disasters, today's technology is to a high extent able to foresee some phenomena and the approximate duration they may last long. However, when it comes to a war situation, an anticipation and prediction of duration it may last becomes very difficult. In reality, all of a sudden, many stakeholder (with political and financial interests) may get involved and a very dynamic atmosphere maybe developed, making it very difficult (if not impossible) to foresee the development even over a very short period of the time. Therefore, any predictive/preventive plan becomes less realistic.

Generally speaking, in case of natural disasters, normally the impacted geographical region is relatively restricted and as a consequence, a reconstruction can be started almost immediately, or at least relatively very quickly in the aftermath of the event and people will be able to return to their residential areas afterwards; while, in the war condition, everything is normally of a different nature. In wars, damages usually are propagated and distributed very quickly across a relatively very wider area. The longer the war lasts, the less becomes the likelihood of any quick return of the refugees back to their homes (those fled out of the region[1]). This is mainly due to the deteriorated economic conditions hindering the reconstruction until

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[1]Here, unless said otherwise, by the word '*refugee*' we refer to those people who had to leave their homes and seek residing elsewhere due to the war condition, independent of being recognized/registered by the UNHCR or not.

a stability is re-established in that political and economic ecosystem. Until then, the UNHCR-recognized refugees are entitled to some supports including humanitarian aids to overcome their essential needs.

A most recent case of this situation is already happening in Syria where a multitude of players have got involved and the situation developed since the unrest in the aftermath of the so called *Arab Spring* has turned into a full-fledged chaotic and asymmetric war spread over a wide region from the south approaching the capital of Iraq, from the east to the boarder villages in Lebanon and up until the Jordanian boarders. According to the UNHCR[2], Syrian war has caused 6.6 million internally displaced persons, 13.1 million people in need inside Syria and 2.98 million people in hard-to-reach and besieged areas. Table 1 reports the statistics reported by the UNHCR[3]. It must be noted that this only concerns the registered refugees and not all-inclusive.

Location Name	Source	Data date	Population	
Turkey	Government of Turkey, UNHCR	18 Oct 2018	63.7%	3,587,930
Lebanon	UNHCR	30 Sep 2018	16.9%	952,562
Jordan	UNHCR	24 Oct 2018	11.9%	672,578
Iraq	UNHCR	30 Sep 2018	4.4%	250,184
Egypt	UNHCR	30 Sep 2018	2.3%	131,504
Other (North Africa)	UNHCR	15 Mar 2018	0.6%	33,545

Table 1: Total Persons of Concern by Country of Asylum.

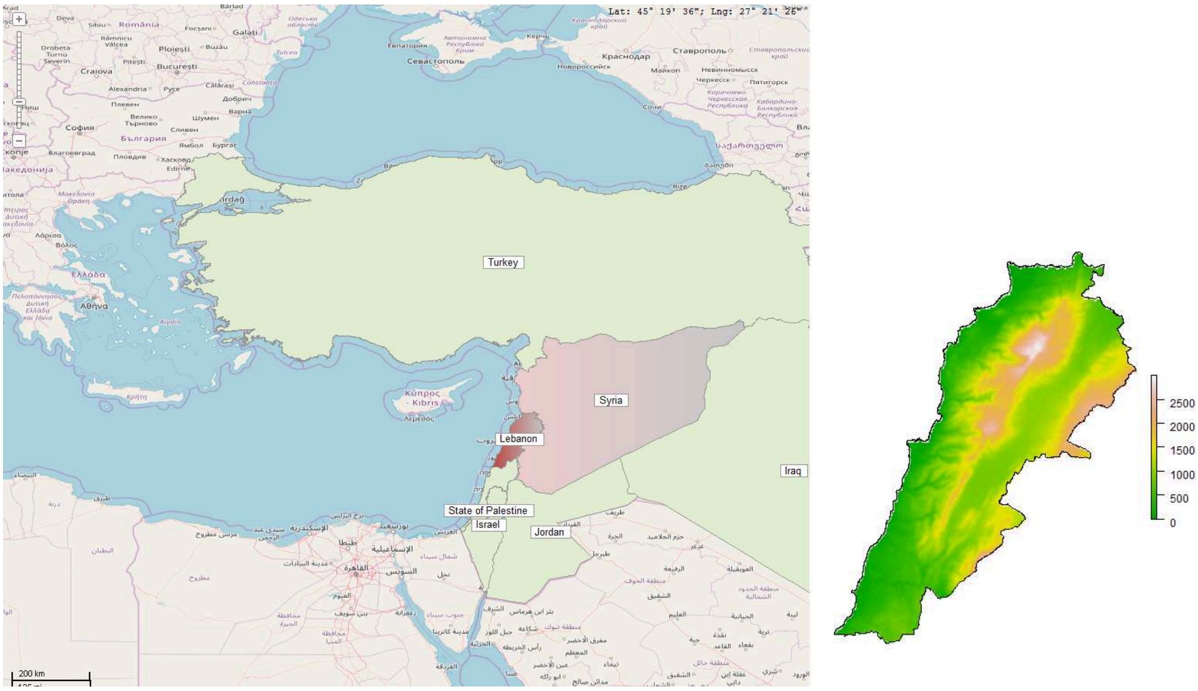
One observes that Lebanon, together with Turkey account for more than 80% of refugees when it comes to the registered refugees. According to the sources within the Lebanon, in 2017, this number reached 1.5 million, which stands for a government estimation accounting for both UNHCR-registered displaced Syrians and non-registered ones.

Lebanese Republic is a country of 6.082 million population with almost half of its borders faced to the Mediterranean Sea and neighbor with two countries Figure 1a. Neither being located in the most peaceful neighborhood in the world nor its political conditions are the most stable one around the globe. The main mode of transport is road for domestic transportation and due to the geographical topology and environmental barriers and conditions, the country does not run any known railway system. Although the country's dimensions are quite limited to an area of 10,452 km^2, the hilly nature of territory causes long travel time between pairs of theoretically close locations and therefore imposing accessibility issue for distribution needs in many cases. Figure 1b depicts elevation plan across this territory.

Since the start of war in Syria, the population of Lebanon has increased by a factor of 25.6% by Syrians, 0.5% by the Palestinian refugees from Syrian and 0.6% by the Lebanese returnees from Syria. In Lebanon, according to the UNHCR, life is a daily struggle for more than a million Syrian refugees, who have little or no financial resources. Around 70% live below the poverty line. There are no formal refugee camps and, as a result, Syrians are scattered throughout more than 2,100 urban and rural communities and locations, often sharing small basic lodgings with other refugee families in overcrowded conditions[4].

Lebanese authority have divided Lebanon into 26 districts at the centroid of the circles depicted in Figure 2a. The circles diameters are proportional to the population of refugees residing in those regions. Historically, for the almost half a million Palestinians living in camps in Lebanon, there has been a number of Distribution Centers (DC) serving those camps across the country. In Figure 2b, the centroid represent

[2] http://www.unhcr.org/syria-emergency.html
[3] https://data2.unhcr.org/en/situations/syria
[4] http://www.unhcr.org/syria-emergency.html



(a) Lebanon and its neighborhood.

(b) Map of elevation.

Figure 1: Geographical condition of Lebanon.

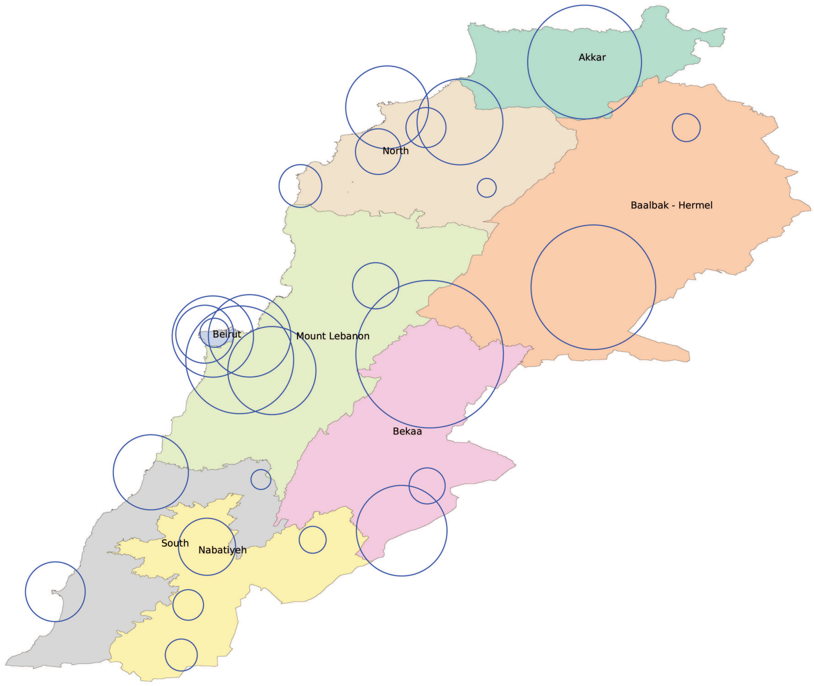
# days	# percentage of show-up
1	100%
2	50% and 50%
3	38%, 24% and 38%
4	31%, 21%, 14% and 34%
5	23%, 18%, 18%, 20% and 21%

Table 2: The user's behavior for different planning horizon lengths.

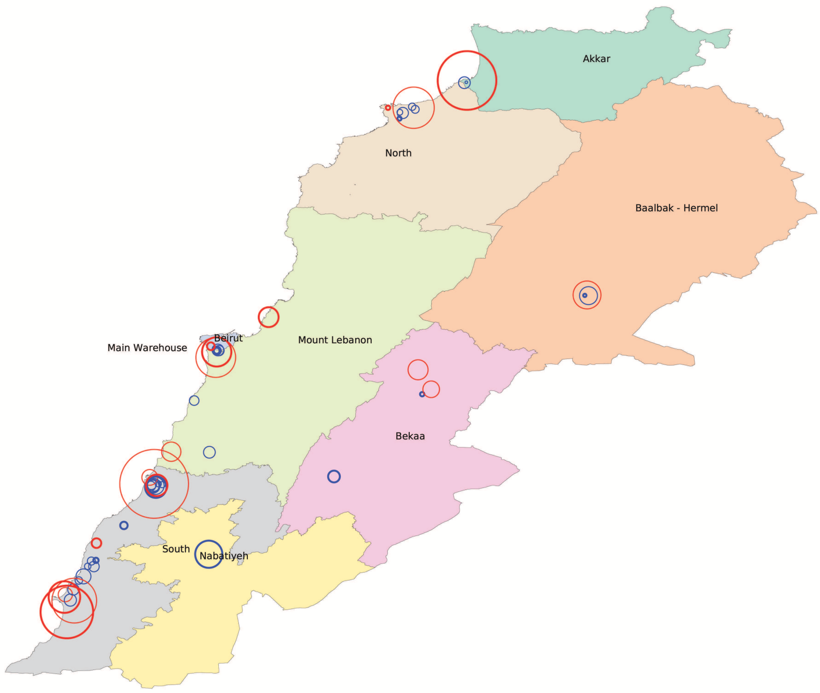
locations of camps and the diameters are proportional to the population accommodated in those regions. The red circles represent the DCs and the main one is in the region of Beirut (annotated in the figure). In the case of Syrian refugees, as in Figure 2a, there are no dedicated DCs.

On the other hand, human factors or user's behavior is another complicating factor, too. The delivery packages of humanitarian aids for a given district usually takes place within a certain interval of time say a week (five working days). A few loaded trucks leave the Main Warehouse (MW) to the districts on a daily basis. While the number of people to be served –during the planning horizon– in each zone is known in advance (due to the known historical data, with a good precision), the number of people showing up to collect their packages depends on the day of distribution. Empirical studies shows that if a 3-day distribution plan is considered, the first and last days are the days with highest percentage (around 38%) of people showing up while the second day attracts the minimum percentage (around 24%) of refugees showing up to collect their packages (see Figure 3). The demand pattern for various planning horizon length is depicted in Table 2.

In general, there is a Main Warehouse (MW), which acquires the nutrition and other essential requirements, needed for developing packages in the form of baskets of given amount of calories, to be sent to the consumers. The preparation of packages are to be done in MWs. Once the packages are formed, depending on the distribution plan, they are loaded on a fleet of trucks and LGVs and are sent to the selected regional



(a) Spatial distribution of Syrian refugees across the Lebanese territory.



(b) Spatial distribution of Palestinian refugees camps in Lebanon.

Figure 2: Spatial distribution of Syrian and Palestinian refugees and the Distribution Centers (DCs).

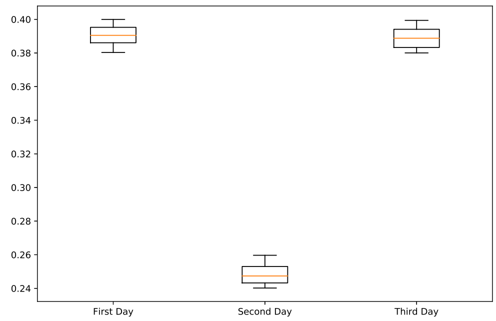


Figure 3: User behavior in collecting the packages of humanitarian aids from the allocated DCs.

DCs. The regional DCs are represented by hub nodes in the model. The next morning, trucks start from the regional DCs and from there, they are deployed to selected points of delivery (the delivery points, other way of saying the refugees camps, are the nodes which are represented by the "spoke nodes" in the model). The delivery points (spoke nodes) are to be visited in a sequential order. every unloaded truck will be discharged from the rest of the operation/itinerary and will leave the fleet immediately. At the end of the day, there must not be any truck left fully or partially loaded. It must be noted that among the locations to be served (the centroid) there are locations with access issues for the trucks; therefore such points are served directly from the DCs using chartered services through a third-party service provider. These points are represented by "isolated nodes" in the model. Once a node received its services (in one, two or three days, depending on the planning), it will no more be considered as a potential spoke(or spur) node until the end of the current planning horizon.

Given the above representation, the problem and the targeted operation fits in with the framework of multi-period Hub-and-Spoke network structures. The objective is to identify the location of a subset of hub nodes (DCs) in each period of the horizon and establish a network with a multi-level structure. The demand of every node at each period is a percentage of the whole demand during the entire planning horizon and depends on the number of antecedent visits to the same node during the same planning horizon. The flows are unidirectional, only from the Main Warehouse (MW) to the final destinations. The fleet of transporters is assumed to be homogeneous.

1.1. Literature review

The hub-and-spoke structures have their roots in the seminal work of Hakimi (1964) –which has then been generalized by Goldman (1969)– for finding the optimal location of a single switching center proven to be on the vertex median of the graph.

The earlier models assume that every origin-destination (O-D) path includes no more than two hub nodes (O'Kelly (1986a,b) and O'Kelly (1987)). In addition to the single or multiple allocation schemes and simpler capacity settings, over the time, further realistic features were integrated into the initial models aiming at achieving a better approximation of real-life practice. At the same time, introducing more realistic features (including multi-modal services) resulted in many more additional variables and constraints and much more complex models, which motivated wider researches including the algorithmic aspects of problems.

While in the earlier contributions, branch-and-cut and enumeration-base methods (see Aykin (1994), Ernst and Krishnamoorthy (1996), Klincewicz (1996), Abdinnour-Helm and Venkataramanan (1998), Ernst and Krishnamoorthy (1999), Ebery et al. (2000), Ernst and Krishnamoorthy (1998b), Mayer and Wagner (2002), Marín (2005a), Sasaki et al. (1999), Sasaki and Fukushima (2003), Labbé et al. (2005), Klincewicz (2002), Campbell et al. (2003), Campbell et al. (2005b) and Yaman and Carello (2005)), Lagrangian-based methods (see Pirkul and Schilling (1998), Elhedhli and Hu (2005), Aykin (1994) and Marín (2005b)), or dual ascent techniques (see Klincewicz (1996), Mayer and Wagner (2002), Cánovas et al. (2007), Contreras et al. (2011d) and Wagner (2007) were the dominant approaches, as the complexity of models has augmented, Benders decomposition paradigm became a very popular technique given their historical success in problems with network design substructures. Perhaps, Camargo et al. (2008) was the first Benders method proposed for an Uncapacitated Multiple Allocation Hub Location Problem. The proposed formulation in this work is referred by other very efficient works afterwards (Gelareh and Nickel (2011), Rodriguez-Martin and Salazar-Gonzalez (2008) and Contreras et al. (2011a)).

1.1.1. Multi-period hub location problem

Very few multi-period hub location models have been proposed in the literature. In this subject, Campbell (1990) proposed a continuous approximation model for a general freight carrier serving a fixed region with an increasing density of demand. Also, Gelareh (2008); Gelareh et al. (2015a) proposed a HLP model within which the transport service provider starts with an initial configuration of the hub-level structure at the first period of the time and then the network evolves in the course of a considered planning horizon. Another study is Contreras et al. (2011b), in which a dynamic uncapacitated hub location problem is studied.

Recently, Monemi et al. (2017) proposed a hub location problem in a co-opetitive setting.

Kara and Taner (2011).

1.1.2. Demand structures in hub location problems

In what concerns the demand structure, Miranda Junior et al. (2011) studies demand uncertainty and congestion with applications tailored for air transport. Also in this context, general demand uncertainty is studied in Alumur et al. (2012) and Contreras et al. (2011c). Later, Merakli and Yaman (2017) and Merakli and Yaman (2016)) were working on introducing robust solution algorithms in presence of various types of demand uncertainty. O'Kelly et al. (2015) analyzes the quality of service in presences of price sensitive demands and proposes relevant models for this variation of the HLP. The literature is unaware of any contribution dealing with a deterministically structured demand patterns or a demand structure resulted by human factors.

O'Kelly and Bryan (1998) dealt with the flow-dependent costs while congestion cost was considered by de Camargo and Miranda (2012); De Camargo et al. (2011); de Camargo et al. (2009a). Rodríguez-Martín et al. (2014) proposed a model for the hub location and routing problem; while Correia et al. (2014) dealt with multi-product and capacity and Sasaki et al. (2014) with a competitive hub location based on Stackelberg games.

1.1.3. Humanitarian subjects in Hub Location Problems

While literature of humanitarian logistics has a rich body, we can only cite a few somehow more relevant ones to our work. In fact, most of contributions concern technical aspects of urban or civil applications that are less relevant to the problems we described here. Yet in the following, we highlight the most recent development in this topic which, are still to some extent relevant.

In humanitarian logistics context, Dufour et al. (2018) analyzes the potential cost benefits of adding a regional distribution center in Kampala, Uganda, to the existing network of the UNHRD in order to better respond to humanitarian crises in East Africa. They carry out a simulation, network optimization and statistical analyses to assess the costs of prepositioning high-demand non-food items in Kampala and to propose a robust stocking solution.

Tofighi et al. (2016) proposed a two-echelon humanitarian logistics network design problem involving multiple central warehouses and local distribution centers and developed a novel two-stage scenario-based possibilistic-stochastic programming approach. What they proposed is a trade-off between the egalitarian and utilitarian objectives. Vahdani et al. (2018) proposed a two-stage multi-objective location-routing-inventory model for humanitarian logistics network design under uncertainty taking into account time factor and robustness of the final solutions under uncertainty. An et al. (2015) studied the hub and spoke network design problem while focusing on the reliability of the service in the network. To this, the authors considered the concept of backup hubs in the phase of planning the network. Zhalechian et al. (2018) proposed a risk analysis study and worked on the hub-and-spoke network design under operational and disruption risks. Later in 2019, Mohammadi et al. (2019) studied the Reliable single-allocation hub location problem while considering disruptions and proposed a bi-objective model for this problem.

Özdamar and Ertem (2015) reviewed the literature on response and recovery planning phases of disaster management and focused on the classification and computational perspectives of humanitarian logistics models highlighting potential research direction. Boonmee et al. (2017) examines the four main problems highlighted in the literature review: deterministic facility location problems, dynamic facility location problems, stochastic facility location problems, and robust facility location problems. For each problem, facility location type, data modeling type, disaster type, decisions, objectives, constraints, and solution methods are evaluated and real-world applications and case studies are presented and at the end the research gaps are being identified.

Elçi and Noyan (2018) proposed a novel risk-averse stochastic pre-disaster relief network design problem. In Elçi et al. (2018) they considered a trade-off between the actual cost and the cost of reliability and proposed a tight MIP formulation.

Charles et al. (2016) proposed a tooled methodology to support humanitarian decision makers in the design of their supply chains. Based on the concept of aggregate scenarios to reliably forecast demand using past disaster data and future trends, some scenarios are generated and demand for relief items based on these scenarios is then fed to a MIP model in order to improve current supply networks.

1.2. Contribution and scope

The prime focus of this paper is to propose a network structure for distribution of humanitarian aids in the case of Syrian refugees in Lebanon. The objective of this work is mainly improving the quality of services provided by the UNHCR, by assuring the frequency of service and receiving the demands in time. Besides, another motivation and objective of this wirk is to propose an exhaustive planning of distributing the demand in the network while optimising the use of the capacity of available resources and also improving the benefits of service providers, in terms of reducing the cost of service provision and the use of navigation fleet (mainly by optimising the use of the capacity of transporters in every trip). This work has been inspired by a thorough investigation of the matter and a close collaboration with the relevant decision makers of the sector. From the theoretical and modeling point of view, this model generalized the previous models of Hub Location Problem by introducing a Multi-Period Hub Location Problem with Serial Demands (MPHLPSD) wherein the demand volume at the n-th visit is proportional to the n-th term of a sequence of number representing human behavior. We propose the first compact mixed integer programming formulation for this problem and show that if Benders decomposition is seen from the perspective of a modeling tool, $\mathcal{O}(n^3)$ variables can be dropped from the formulation, in addition to the constraints being projected out in Benders fashion. Several classes of effective valid inequalities and efficient separation routines are proposed in order to turn the solution approach into a very efficient cut-and-Benders method reporting a significantly accelerated convergence. For the case at hand, we report the optimal location of DCs to be used in Lebanon. An extensive computational experiments on randomly generated instances of various sizes, confirms computational efficiency of the proposed solution framework and viability of technique.

This paper is organized as follows: The problem is formally described in Section 2 and a mathematical model together with several classes of valid inequalities is proposed in Section 3. Section 4 describes and elaborates on the design of our solution approach and the associated components. Section 5 reports some computational experiments and gives insights into the effectiveness of the proposed approach. In Section 6, we summarize, draw conclusions and provide suggestions for further research directions.

2. Problem Description

A Main Warehouse acquires the nutrition and composes packages to be distributed, among refugees at the demand points. Every demand point is served q times (at most 3 times, in practice) during a distribution horizon of length T (normally a week, i.e., 5 days) where $q < T$. For every demand node that is actually representing a refugee's camp in the network, the sum of demands collected by the associated refugees over the distribution days is an exogenous part of the problem and equivalent to the total demand of that node. Refugees show up at the collection points following a known pattern as a sequence $\{l_p\}_{p \in i,\ldots,m}$ wherein $\sum_1^m l_p = 1$, i.e., $l_p \times 100$ percent of the refugees settled in this point (the current visiting camp) are showing up at the $p-$th visit to this point during the planning horizon. Each day, a subset of all demand nodes are chosen to be visited and served by the distributors. These nodes are then partitioned into groups/clusters of ordered nodes, according to the order by which they are going to be served. One node per cluster is designated as a DC. Due to some known reasons including geographical barriers and/or security issues, certain locations in a cluster cannot be served on the route. These nodes are referred to as *isolated* nodes. A convoy of trucks transporting the packages travel overnight from the MW towards a DC node in every cluster, which has the operational capacity of accommodating and assuring the security of the convoy of trucks. We assume that the fleet is a homogenous one as all transporters have comparable specifications. The journey starts the day after, where the DC node is served first and then all the other nodes according to the order and at the end the tour terminates when the last node on the route is served. At every DC node at which an isolated one is allocated, a shuttle service load the demand of isolated node (sufficient supply according to the p-th visit's demand of that node) and departs from the DC. Each time a demand point is served (its planned daily demand is delivered), the fully unloaded trucks, if any, are discharged and will leave the convoy. A total length of travel from a DC visiting all nodes sitting on the route, up until the last one must not exceed a pre-defined number of nodes as the convoy wishes to finish the task before the end of day (roughly speaking, the sunset time). A demand node (representing a camp) might have already been visited q times on the days before the current day, while in such a situation, it will no more be visited on the current day or afterwards. If not, it may be visited on the current day of the horizon, or if there is not a visit planned for it on the current day, it will have a visit (or more, if needed) on the forthcoming days before the end of the planning horizon, to ensure the exactitude of receiving q visits before the end of the horizon. It is to clarify that on a certain day, a demand node may have a visit or not, while it definitely have q visits on different q days during the distribution horizon to cover the total demand of it.

We seek at every period, locating an optimal number of DCs and identify the demand nodes to be visited, while avoiding them being visited more than $q = 3$ times in a period, the clusters to be formed, the sequences of demand point being served by every DC (on the route or directly using a chattel service) and the optimal number of transporters, such that the total cost of operations accounting for the fixed set up cost for DCs and the total transportation and fixed vehicles costs is being minimized.

In the remainder of this paper, we will use DC and hub node, demand point and spoke node interchangeably.

3. Mathematical Model

The problem as described in the previous section resembles a hub-and-spoke structure and gives rise to a new variant of this problem, which to the best of our knowledge, has not been studied previously. In this section, we present a MIP formulation for this problem and will refer to it as *Multiperiod Hub Location Problem with Serial Demand (MPHLPSD)*. The parameters are listed in Table 3.

c^t_{ij}:	the unit transportation cost associated with the arc (i,j) at period t,
w_j:	the demand at node j,
b:	the penalty cost coefficient applied to the arc connecting an isolated node,
F^t_i:	the fixed cost of using location i as a DC in period t,
$\mathscr{N}^0$:	$\mathscr{N} \cup \{0\} = \{1,\ldots,N\} \cup \{0\}$ represents the set of nodes acting as DCs; 0 being the Main Warehouse,
V^t_{0i}:	the fixed cost of deploying a truck from the MW to the DC located at i,
$\mathscr{T}$:	$\mathscr{T} = \{1,\ldots,T\}$ represents the planning horizon, where $\lvert\mathscr{T}\rvert = T$ is the length,
$\mathscr{P}$:	$\mathscr{P} = \{1,\ldots,q\}$ represents the set of call numbers to every node, where $\lvert\mathscr{P}\rvert = q$ is the visit frequency,
$\mathscr{DC}$:	$\mathscr{DC} \subseteq \mathscr{N}$ represents the set of potential DC nodes,
N^{min}/N^{max}:	represent the min/max number of nodes (spoke nodes) allocated to every DC in a given period,
V^{cap}:	the typical capacity of vehicles in a convoy,
l_p:	the fraction of demand to be delivered during the p-th visit.

Table 3: Model parameters.

3.1. *Decision variables:*

Decision variables required to model this problem are listed below:

r_{ijt} is 1, if arc (i,j) is an arc belonging to one of the service routes at period t, 0 otherwise.
z^t_{ij} is 1, if route i is allocated to hub j in period t, self-allocation represents a DC at i. It means that z^t_{ii} is 1, if i is a DC node at period t, and 0 otherwise.
y^t_{ii} is 1, if i is an isolated node at period t, and 0 otherwise.
y^t_{ij} is 1, if j is an isolated node in period t allocated to a DC i, and 0 otherwise.
ζ^p_{it} is 1, if p-th visit to i takes place at period t, and 0 otherwise.
x^t_{jkl} represents the fraction of flow from the MW to j traversing arc (k,l) in period t.
And finally v^t_{kl} represents the number of transporters deployed on the arc (k,l) in period t.

We also introduce $G(V, A \cup A')$, $S \subset V$, $\delta^+(S) = \{a = (i,j) | i \in S, j \in V/\,S\}$, $\delta^-(S) = \{a = (j,i) | i \in S, j \in V/\,S\}$ and $\gamma(S) = \{a = (i,j) | i,j \in S, j \neq i\}$.

3.2. *A mixed integer linear programming model*

We propose the following MIP for the MPHLPSD:

(MPHLPSD)

$$\min \sum_t \sum_j \sum_{k \notin j,l} w_j c^t_{kl} x^t_{jkl} + \sum_t \sum_{k \neq MW : MW = 0} V^t_{0k} v^t_{0k} + \sum_t \sum_{i,j \neq i} b c^t_{ij} y^t_{ij} + \sum_t \sum_{i : i \neq MW} F^t_i z^t_{ii} \quad (1)$$

s. t.

$$N^{min} \leqslant \sum_j z^t_{ji} \leqslant N^{max}, \quad \forall i,t, \quad (2)$$

$$\sum_{j \neq i} r^t_{ij} = \sum_j z^t_{ij}, \quad \forall i,t, \quad (3)$$

$$\sum_{j \neq i} r^t_{ji} = \sum_j z^t_{ij}, \quad \forall i,t, \quad (4)$$

$$r^t_{ji} + r^t_{ij} \leqslant 2 - z^t_{ik} - z^t_{jl}, \quad \forall i,j,k,l,t,k : j \neq i, k \neq l, \quad (5)$$

$$\sum_t (\sum_{j \neq i} z^t_{ij} + z^t_{ii} + y^t_{ii}) = q, \quad \forall i,t, \quad (6)$$

$$r^t_{ij} + r^t_{ji} \leqslant 1, \qquad \forall i, j, t : j \neq i, \quad (7)$$

$$\sum_t \zeta^p_{it} = 1, \qquad \forall p, i, \quad (8)$$

$$\zeta^1_{i1} = z^1_{ii} + \sum_{j \neq i} z^1_{ij} + y^1_{ii}, \qquad \forall i, j : j \neq i, \quad (9)$$

$$\zeta^p_{it} \leqslant z^t_{ii} + \sum_{j \neq i} z^t_{ij} + y^t_{ii}, \qquad \forall i, j, p, t : j \neq i, t \geqslant 2, \quad (10)$$

$$\sum_{t'=1}^{t} \zeta^{p+1}_{it'} \leqslant \zeta^p_{it}, \qquad \forall i, t, p : p \geqslant 1, \quad (11)$$

$$z^t_{ij} \leqslant z^t_{jj}, \qquad \forall i, j : j \neq i, t, \quad (12)$$

$$\sum_{j \neq i} y^t_{ji} = y^t_{ii}, \qquad \forall i, t, \quad (13)$$

$$r^t_{0i} = z^t_{ii}, \qquad \forall i, t, \quad (14)$$

$$y^t_{ii} \leqslant \sum_p \zeta^p_{it}, \qquad \forall i, t, \quad (15)$$

$$y^t_{ij} \leqslant z^t_{ii}, \qquad \forall i, j, t : j \neq i, \quad (16)$$

$$\sum_{j \neq i} y^t_{ij} \leqslant 1, \qquad \forall i, t, \quad (17)$$

$$\sum_p \left(\zeta^p_{it} + \zeta^p_{it+1} + \zeta^p_{it+2}\right) \leqslant 2, \qquad \forall i, t \in \{1, N-2\}, p, \quad (18)$$

$$\sum_{k \notin i,0} w_j x^t_{j0k} = w_j \sum_p l_p \zeta^p_{jt}, \qquad \forall t, j :\neq MW, \quad (19)$$

$$\sum_{k \neq j} w_j x^t_{jkj} = w_j \sum_p l_p \zeta^p_{jt}, \qquad \forall t, j \neq MW, \quad (20)$$

$$\sum_{l \neq 0} w_j x^t_{jkl} = \sum_{l \neq k, k \neq 0} w_j x^t_{jlk}, \qquad \forall t, j : j \neq MW, k \notin \{j\}, \quad (21)$$

$$x^t_{j0l} \leqslant r^t_{0l}, \qquad \forall j, l, t, \quad (22)$$

$$x^t_{jkl} \leqslant r^t_{kl} + y^t_{kl}, \qquad \forall t, j, k : k \neq j, l \neq k, \quad (23)$$

$$x^t_{jkl} \leqslant \sum_j z^t_{kj}, \qquad \forall t, j, k : k \neq j, l \neq k, \quad (24)$$

$$\sum_k w_j x_{j0kt} \leqslant V^{cap} v^t_{0k}, \qquad \forall k, t, \quad (25)$$

$$x^t_{jkl} \in (0,1)^{|\mathcal{N}^0|^3|T|}, r^t_{ij}, z^t_{ij} \in \{0,1\}^{|E||T|}, \zeta^p_{it} \in \{0,1\}^{pN|T|}, v^t_{kl} \in \mathbb{N}^{N|T|}. \quad (26)$$

The objective function (1) accounts for the total transportation cost as well as the cost of deploying transporter on legs of call, fixed setup cost for the DCs and the penalty cost of having isolated nodes.

The number of nodes along every route is bounded between a minimum and a maximum number to avoid too many stops as per constraint (2). Exactly one arc arrives to/leaves a node that belongs to a route. This is expressed by the constraints (3)-(4). A route arc can only be established between two spoke nodes allocated to the same hub nodes in the same period as guaranteed in constraints (5). Constraints (6) ensure that q times during the distribution horizon every node is being served —either directly or indirectly. Two arcs (i, j) and (j, i) cannot be established simultaneously between two nodes i and j. This is guaranteed in constraints (7). Every node has a first, second, ..., p-th time that it has been visited as per constraints (8). A visit to a node at the first period corresponds to the first visit as stated in constraints (9). For every p-th visit to a node in a given period, the node must appear in that period, as stated in constraints (10). Constraints (11) ensure that the $p+1$-th visits cannot take place before the p-th one. Constraints (12) ensure that every route node must be allocated to a hub node. Every isolated node must be connected to the rest of the network via an arc that arrives to it from one route node (hub or non-hub) from among all of them. This is ensured by constraints (13). At every period, there is an arc from the MW to every hub node as stated in constraints (14). Constraints (15) ensure that an isolated node of a given period will not be opened unless the number of visits to that node is incremented. Constraints (16) ensure that the tail node of an arc from a route node towards an isolated node must be a DC. A DC node cannot serve more than one isolated node as per constraints (17). Constraints (18) ensure that no point is visited $q = 3$ times in a row (q consecutive periods). The volume of flow originated from the MW and destined to every demand node corresponds to the demand pattern in the p-th visit. Constraints (19)- (21) are the flow conservation constraints. At every period, there is an arc encompassed from MW to every designated hub nodes as stated in constraints (22). Constraints (23) ensure that flow from the MW to the DCs traverse existing arcs. Constraint (24) ensure that if k is not MW, flow does not enter any arc (k, l) unless k is a ring node (unless k belongs to the same ring that l belongs to it). The number of trucks leaving from the MW is sufficient to accommodate the volume being shipped. This is guaranteed by constraints (25).

3.3. Illustrative Example

In Figure 5, the evolution of network over a period of one week is depicted. The planning horizon is one *week* and every node is served exactly 3 times. It must be noted that this is an artificial example assuming all nodes can be DCs or isolated nodes. In this figure, arrows represents the suggested routs. Dashed arrows represent routs to serve iusolated nodes by a direct shuttle and dotted nodes shows where a truch is unloaded and from that node, it discharged from the service and directly returned to the attributed DC. The rout does not end there, since there are other trucks in the fleet that continues their itinerary to finish their assignments.

3.4. Valid Inequalities

We introduce $r(\delta(S)^+) = \sum_{i\in S, j\in V/\ S} r_{ij}$, $r(A(S)) = \sum_{(i,j)\in A(S)} r_{ij}$, where $A(S) = \{a = (i,j) \in A : i,j \in S\}$ and $r(P) = \sum_{(i,j)\in P} r_{ij}$ where $P = \{(i_0, i_1), (i_1, i_2), \ldots, (i_{k-1}, i_k)\}$.

Proposition 1. *Let V be the set of nodes in the network. The following inequalities are valid for (MPHLPSD):*

$$r^t(A(S)) \leqslant |S|, \qquad S \subset V \tag{27}$$

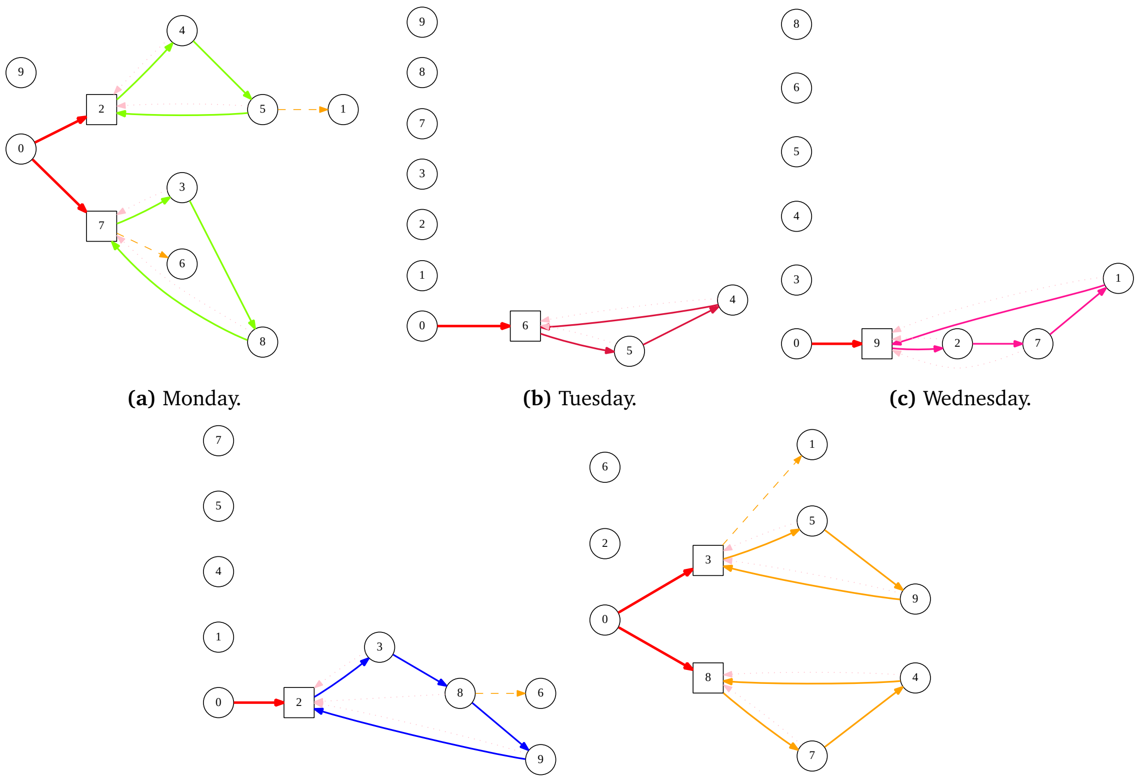
Proposition 2. *If $i \in S \subset V$ and i is a non-isolated demand node served by a DC in $V/\ S$ at period t, there is at least one arc going out of (entering into) S:*

$$a)\ \ r^t(\delta^+(S)) \geqslant \sum_{j\in V/\ S} z^t_{ij}, \qquad \forall i \in S \subset V, t, \tag{28}$$

$$b)\ \ r^t(\delta^-(S)) \geqslant \sum_{j\in V/\ S} z^t_{ij}, \qquad \forall i \in S \subset V, t. \tag{29}$$

Proof. a) For a given $S \subset V$ and any $i \in S$, in the absence of any arc (i, j) with the tail in S and the head in the complementary set, any path from i to the DC serving i entirely lies within S. Thus, there exists no j outside S, which serves i for any choice of $j \in V/\ S$. On the contrary, if i is served by a DC in $V/\ S$ there is a unique path cutting S (at least once) towards the DC serving i.
b) Analogous to the proof of a). □

See Monemi and Gelareh (2017) and Martín et al. (2014) for similar valid inequalities.



(a) Monday. **(b)** Tuesday. **(c)** Wednesday.

(d) Thursday. **(e)** Friday.

Figure 4: An illustration of network structure over the period of week. Every node is visited $p = 3$ times. The nodes on the left side of images correspond to the ones that were not served in the given period. The nodes 2 and 7, which were DCs in the first day, do not appear on the network of Tuesday and on Wednesday, they become normal demand points being served by the node 9 as DC to which they are allocated. While on Monday and Thursday, the node 6 is an isolated node (perhaps) it becomes a DC node on Tuesday.

Proposition 3. *In period* t*, if* $i \in S \subset V$ *and* $i' \in V/\,S$ *be two demand nodes where* i *is served by a DC in* $V/\,S$ *and* i' *served a DC in S, we have:*

$$r^t(\delta^+(S)) \geqslant \sum_{j \in V/\,S} z^t_{ij} + \sum_{j' \in\, S} z^t_{i'j'}, \qquad \forall i \in S \subset V, i' \in V/\,S,\ t, \tag{30}$$

$$r^t(\delta^-(S)) \geqslant \sum_{j \in V/\,S} z^t_{ij} + \sum_{j' \in\, S} z^t_{i'j'}, \qquad \forall i \in S \subset V, i' \in V/\,S,\ t. \tag{31}$$

Proof. Similar to 2. See Monemi and Gelareh (2017) and Martín et al. (2014) for similar case in different problems. □

Proposition 4. *If* $(i-j)$ *or* (j,i) *is an arc in period* t *and* j *is served by DC* k *then* i *must be also be served by* k*:*

$$r^t_{ij} + r^t_{ji} + z^t_{jk} \leqslant 1 + z^t_{ik} \qquad \forall i,j,k : j \neq i,\ t. \tag{32}$$

Proposition 5. *For all* $S \subseteq V,\ t$*,*

$$r^t(A(S)) - \sum_{i \in S} z^t_{ii} \leqslant |S| - \lceil \frac{|S|}{N^{max}} \rceil \tag{33}$$

is a valid inequality.

Proof. Given that $r^t(E(S)) = r^t(A(S))$, c.f. Proposition 2 in Martín et al. (2014). □

The so-called *chain barrier constraints* or *path inequalities* (see Fischetti et al. (1998a)) inequalities are valid inequalities for MPHLPSD.

Proposition 6. *For each* $t, S \subseteq V, \{i,i'\} \subseteq I$ *and a path* $P := \{(i,i_1),(i_1,i_2),\ldots,(i_k,i')\}$ *from* i *to* i'*, the inequality:*

$$r^t(P) + \sum_{j \in S} z^t_{ij} + \sum_{j \notin S} z^t_{i'j} \leqslant |P| + 1 \tag{34}$$

is valid for MPHLPSD polytope. Constraints (5) *are special case of* (34) *when* $P := \{[i,i']\}$*.*

Proposition 7. *If for a given arc in a given period* t*, its tail is in* S *and its head is in* $V\,/S$ *(or vice versa), either the tail is allocated to a hub node in* $V/\,S$ *or the head is allocated to a hub node in* S*. For all* $(i,i') \in A$ *and* $S \subseteq V$ *where* $i \in S, i' \in V/\,S$*,*

$$r^t_{ij} + r^t_{ji} \leqslant \sum_{j \in V/\,S} z^t_{ij} + \sum_{j' \in S} z^t_{i'j'} \tag{35}$$

is a valid inequality.

Corollary 1. *Constraints* (7) *are special cases of constraints* (27) *where* $|S| = 1$

Carroll et al. (2013); Carroll and McGarraghy (2013) defines the *ghost* ring as follows. We establish the support graph of a fractional solution and identify cycles that are longer than the ring bound, if any. We call these infeasible rings *ghosts*. In such cases, at least two edges must be removed from every such ring.

$$\sum_{e \in \gamma(G_r)} r_e \leqslant |G_r| - \max\{\lceil |G_r|/N^{max} \rceil, 2\}, \qquad \forall \text{ Ghost ring } G_r. \tag{36}$$

Proposition 8. *The following inequalities are valid for* $\mathscr{P}(\text{MPHLPSD})$*:*

$$\sum_{a\in\delta^{\diamondsuit}(S)} r_a^t \geqslant \left(\frac{\sum_{i\in S}\sum_{j\neq i} z_{ij}^t + \sum_{a\in\delta^{\diamondsuit}(S)} r_a^t}{N^{max}} - \sum_{i\in S} z_{ii}^t \right) \tag{37}$$

for all $S \subset V$*. This delivers a lower bound on the number of edges encompassed from a set* $S \subset V$*, where* $\diamondsuit \in \{+,-\}$*.*

Proof. See the proof in Martín et al. (2014). □

Proposition 9. *The following 2-matching constraints for are valid for* $\mathscr{P}(\text{MPHLPSD})$ *in every period* t*:*

$$\sum_{a\in\gamma(H)} r_a^t + \sum_{a\in\mathscr{T}} r_a^t \leqslant \sum_{i\in H}\sum_{j\neq i} z_{ij}^t + \lfloor\frac{|\mathscr{T}|}{2}\rfloor \tag{38}$$

for all $H \subset V$ *and all* $\mathscr{T} \subset \delta^{\diamondsuit}(H)$ *satisfying,*
i) $|\{i,j\}\cap H| = 1, \quad \forall\{i,j\}\in\mathscr{T}$*,*
ii) $|\{i,j\}\cap\{k,l\}| = \emptyset, \quad \{i,j\}\neq\{k,l\}\in\mathscr{T}$*, and*
iii) $|\mathscr{T}| \geqslant 3$ *and odd.*

where $H \subset V$ *is a handle and* $\mathscr{T} \subset \delta^{\diamondsuit}(H)$ *are called teeth, where* $\diamondsuit \in \{+,-\}$*.*

Proof. In every feasible solution of this problem we have,

$$2r^t(\gamma(H)) + r^t(\delta^+(H)) + r^t(\delta^-(H)) = \sum_{i\in H}(r^t(\delta^+(i)) + r^t(\delta^-(i)))$$

and from constraints (3) and (4) one obtains:

$$\sum_{k\in H}\sum_{l\neq k}(r_{kl}^t + r_{lk}^t) \leqslant 2\sum_{k\in H}(\sum_j z_{kj}^t).$$

Hence,

$$\begin{aligned} 2\sum_{k\in H}(\sum_j z_{kj}^t) &\geqslant 2r^t(\gamma(H)) + r^t(\delta^+(H)) + r^t(\delta^-(H)) \\ &= 2r^t(\gamma(H)) + r^t(\delta^+(H/\mathscr{T}) + r^t(\delta^+(\mathscr{T})) + r^t(\delta^-(H/\mathscr{T})) + r(\delta^-(\mathscr{T}))). \end{aligned} \tag{39}$$

Given that $r^t(\delta^+(\mathscr{T})\cup\delta^-(\mathscr{T})) \leqslant |\mathscr{T}|$ deduced from the bound constraints $r_a^t \leqslant 1$ and (7), we add this set of constraints to (39) and we obtain:

$$\begin{aligned} 2\sum_{k\in H}(\sum_j z_{kj}^t) + |\mathscr{T}| &\geqslant 2r^t(\gamma(H)) + r^t(\delta^+(H/\mathscr{T}) + r^t(\delta^-(H/\mathscr{T})) + 2r^t(\delta^+(\mathscr{T})) + 2r^t(\delta^-(\mathscr{T}))) \\ &\geqslant 2r^t(\gamma(H)) + 2r^t(\delta^+(\mathscr{T})) + 2r^t(\delta^-(\mathscr{T})). \end{aligned} \tag{40}$$

Again, given that $|\mathscr{T}|$ is an odd number, by multiplying both sides by $\frac{1}{2}$, we conclude,

$$r^t(\gamma(H)) + r^t(\delta^+(\mathscr{T}) + r^t(\delta^-(\mathscr{T}))) \leqslant \sum_{k\in H}(\sum_j z_{kj}^t) + \frac{|\mathscr{T}|-1}{2} \tag{41}$$

and the proof is complete.

□

Proposition 10. *The following comb inequalities are valid for* $\mathscr{P}(\text{MPHLPSD})$*: in every period* t

$$\sum_{a\in\gamma(H)} r_a^t + \sum_{j=1}^{t} \sum_{a\in\gamma(T_j)} r_a^t \leqslant \sum_{i\in H}\sum_{j} z_{ij}^t + \sum_{j=1}^{t} |T_j| - \frac{3t+1}{2}, \tag{42}$$

for all $H \subset V$ *and all* $T_i \subset V$, $\forall i \in \{1, \dots, t\}$ *satisfying,*
i) $H, T_1, T_2, \dots, T_t \subseteq V$,
ii) $T_j \setminus H \neq \emptyset$, $\forall j \in \{1, \dots, t\}$,
ii) $T_j \cap H \neq \emptyset$, $\forall j \in \{1, \dots, t\}$,
ii) $T_i \cap T_j = \emptyset$, $\forall j \in \{1, \dots, t\}$, *and*
iii) $t \geqslant 3$ *and odd.*

Proof. For $S \subset \mathscr{P}$, in every feasible solution we have,

$$r^t(\gamma(H)) = \sum_{i\in H}(\sum_{j} z_{ij}^t) - \frac{r^t(\delta^+(S)) + r^t(\delta^-(S))}{2}, \tag{43}$$

and for every $T_j \subset \mathscr{P}$, $j \in \{1, \dots, t\}$ we have,

$$r^t(\gamma(T_j)) = \sum_{i\in T_j}(\sum_{j} z_{ij}^t) - \frac{r^t(\delta^+(T_j)) + r^t(\delta^-(T_j))}{2}. \tag{44}$$

Furthermore, from (43) and (44), one yields:

$$r^t(\gamma(H)) + \sum_{i} r^t(\gamma(T_i)) = \sum_{i\in H}(\sum_{j} z_{ij}^t) + \sum_{j}\sum_{i\in T_j}(\sum_{j} z_{ij}^t) - \frac{1}{2}\left((r^t(\delta^+(H)) + r^t(\delta^-(H))) + \sum_{j}(r^t(\delta^+(T_j)) + r^t(\delta^-(T_j)))\right). \tag{45}$$

Let $\delta_i^+(H) \cup \delta_i^-(H)$ denote the cut set associated with arcs having one end-point in $H \cap T_i$ and another end-point outside H. We know that $r^t(\delta^+(H)) + r^t(\delta^-(H)) \geqslant \sum_i (r^t(\delta_i^+(H)) + r^t(\delta_i^-(H)))$.

It can be easily shown that $(r^t(\delta_i^+(H)) + r^t(\delta_i^-(H))) + (r^t(\delta^+(T_i)) + r^t(\delta^-(T_i))) \geqslant 3$. We also know that $r^t(\delta^+(H)) + r^t(\delta^-(H))$ and $r^t(\delta^+(T_i)) + r^t(\delta^-(T_i))$ are even numbers and therefore $r^t(\delta^+(H)) + r^t(\delta^-(H)) + \sum_i (r^t(\delta^+(T_i)) + r^t(\delta^-(T_i))) \geqslant 3t + 1$. By substituting in (45) the proof is complete.

□

Proposition 11. *Monemi and Gelareh (2017) proposed the so-called* moving sub-path inequalities *are valid for* $\mathscr{P}(\text{MPHLPSD})$*. Let* r^* *be an integer partial solution and a violated inequality of (36) in the form of* $P = (\nu_1, \nu_2, \dots, \nu_m, \nu_1) : m > R$ *exist. We can identify some minimal subsets of variables (corresponding to a sequence of edges) in such a path for which the ring bound capacity is violated. The following valid inequalities are violated by such a solution:*

$$\sum_{e=\{i,i+1\}:\ i\in\{i_1,\dots,i_1+R\}} r_e^t \leqslant N^{max}, \qquad \forall t, i_1 \in \{\nu_1, \nu_2, \dots, \nu_1 + N^{max}, \dots, \nu_m\} : i_1 + N^{max} \leqslant m - 1 \tag{46}$$

where i *is selected as a rolling sequence basis.*

Proof. See Monemi and Gelareh (2017). □

4. Solution Method

The proposed solution methods can be categorized as a Benders decomposition method. In fact, as it will be shown in this section, a Benders reformulation can help getting rid of $\mathcal{O}(|N||T||P|)$ variables ζ_{it}^{p} as it will implicitly take care of their role and logical implications without any need to have them explicitly present in the model and increase computational complexity. In other words, it will be shown that a Benders reformulation in addition to projecting the model onto the space of integer variable can actually help us to get rid of many integer variables that were otherwise inevitable in a compact formulation.

Benders decomposition (Benders, 1962, 2005) is a primal decomposition method, proven to be a very efficient paradigm in dealing with large-scale MIP models arising in problems within an underlying location/network structures. Benders decomposition is based on projecting the model on the space of complicating variables and exploits the primal/dual relationship with a sub-problem to generate cuts to separate solutions of the master problem and tighten the outer approximation, until optimality is proven as a result of lower and upper bounds converging towards the same value.

If a solution to the master problem represents direction of an improving ray in the subproblem dual, then one can generate feasibility cuts to avoid such solutions from the master problem otherwise, the primal-dual relationship can be exploited to generate optimality cuts and improve the outer approximation, iteratively. Clearly, the first class of cuts are no-good cuts to prevent infeasible solutions of master problem while the second class tend to push the bound(s) and accelerate convergence. As such, some authors have proposed to replace the actual master problem with an auxiliary/alternative ones making sure that no solution of the master problem can point to a ray direction in the subproblem dual. An example of this can be seen in Gelareh and Nickel (2011), which replaces the master problem with a different model in Maculan et al. (2003) delivering only feasible solutions. This has proven to be very efficient, depending on the way Benders is being implemented —traditional or branch-and-cut style. Other work such as those in Codato and Fischetti (2006) and Fischetti et al. (2009) proposed other eliminatory approaches based on *minimal infeasible subsystem* to cut off bad solutions.

Tightening Benders standard cuts in Benders has been considered since 80's where the first efforts were focused on finding non-dominated Benders cuts from among alternative ones resulted by multiple optimality in the subproblem dual. After the pioneering work of Magnanti and Wong (1981) in dealing with degenerate subproblems by sharpening the cuts using *relative interior points*, Papadakos (2008) showed that the same can be done but with less difficulties —no need for the points to in the relative interior of master problem polytope. Furthermore, Papadakos (2008) proposed an iterative algorithm to update this point. Other general techniques have been proposed such as using heuristic solutions to accelerate convergence (see Sherali and Fraticelli (2002), McDaniel and Devine (1977) and Sherali and Fraticelli (2002) for instance).

Earlier work on Benders decomposition for variants of hub location problems include Camargo et al. (2008); de Camargo et al. (2009b) for models with emphasis on economies of scale, Gelareh and Nickel (2011), Gelareh et al. (2015b) for single and multiple period hub location problems and Contreras et al. (2011a) for a classical model of the uncapacitated hub location problem. In the recent years, other successful applications have been reported in the literature.

Some very recent work in literature dealing with Benders technique for variants of hub location problems include Mokhtar et al. (2018) for a 2-allocation p-hub median problem, Rostami et al. (2018) for a single allocation problem under breakdown reliability and Real et al. (2018) for a Gateway Location Problem.

Rahmaniani et al. (2017) presented a sophisticated analytical literature review of the method and addressed most of the important features.

Our proposed framework is a branch-cut-Benders approach. The fractional solutions of master problem and those that result in infeasible network structures are cut using efficiently separated valid inequalities at appropriate stages along the branch-and-cur process. We only deliver integer-feasible solutions (feasible network structures with respect to the whole problem) of MP to the Benders subproblem and we do this by efficiently separating the appropriate valid inequalities –from a portfolio of identified ones– to cut the infeasible solution of master problem. In addition, we use an inexpensive heuristic algorithm to generate feasible solutions that warm-start our solution algorithm with an optimality cut corresponding to an integer feasible solution of the heuristic.

4.1. Heuristic solution

Poojari and Beasley (2009) proposed to use meta-heuristics to find feasible solutions, generate Benders and add to the MP before Benders process being started hoping that this warm-start would accelerate the convergence of algorithm. Our extensive computational experiments on a wide range of problems revealed that this strategy is particularly efficient when the original compact formulation is very tight, otherwise, even by supplying the optimal solution one may not see a significant gain in the additional overhead of running this heuristic. Another interesting finding is that, in network design problems with less dense or even sparse flow matrices, this only generates overhead (again this conclusion is limited to the benchmark of our extensive computational experiments). Therefore, it is always wiser to find a trade-off between the quality of solution we expect from the heuristic and the time spent on it. Our extensive computational experiments on a variety of models let us conclude that this heuristic procedure should remain very inexpensive.

The proposed minimalist local search is the following: Given constraints (18), we serve all nodes in periods 1 and 2, then skip the third period and again repeat this pattern until every node is visited q times. Then in every period, the non-MW nodes with demand volume in the upper $\frac{N}{N^{min}}$ are chosen to be the DCs. The remaining nodes are allocated to the DCs based on the distance (ties are broken in favor of having balanced clusters with respect to the volume of demand) creating routes with N^{min} nodes along them. The remaining $N - \lfloor \frac{N}{N^{min}} \rfloor \times N^{min}$ nodes are added one by one to every DC's cluster. The routes are then constructed by connecting nodes of every cluster.

The efficiency in computing the objective function allows us to do certain number (practically, we fixed it to 10000) of iterations for improving the initial solutions. There neighborhood are defined as in the following: (1) reordering the nodes within every route; (2) moving a whole route of a given DC to the periods where no distribution takes place, (3) let any other node in the cluster be the DC and turn the DC to a normal route node, and (4) if there are more than q period within which distribution takes place, a node can move from one period to another period and in any route there. We start the local search with the neighborhood (1) and upon encountering $2N$ non-improving solutions we move to the next one with the order defined by the number associated to every neighborhood. One may think of it as a naive version of a Variable Neighborhood Search (VNS) (see Hansen et al. (2008) for a general review of the method and Freitas and Penna (2019), Herrán et al. (2019) and Chagas et al. (2019) for instances of recent successful application) even though it not intended to be a full-fledged and mature heuristic or a stand-alone solution methods producing high-quality solutions.

4.2. Initial relaxation

We have chosen the constraints (2), (3), (4), (6), (7), (12), (13), (14), (15), (16), (17) and (18) to constitute the initial relation (or the master problem (MP)). Constraints (5) are excluded and be separated upon need, because in spite the large cardinality of this set of constraints which is $\mathcal{O}(N^4T)$ and deteriorates the performance of solvers very significantly, only a very tiny fraction of them may be active in an optimal (or any feasible) solution of the problem.

$\zeta^p_{it}, \forall p, i, t$ can be totally eliminated from the MP as these variables and all related constraints are no longer required and their presence would be only imposing additional computational burden (branching, reduction, etc.) on the shoulders of the MIP solver solving MP. In fact, once the set of DCs and demand nodes are determined for every period t, the d-th days at which every node is visited becomes known (*a posteiori*) and this information can be observed by the subproblem through the solution provided by MP. Subproblem can then determine the volume of demand to be supplied to every demand point given the percentage corresponding to the p-th visit to the node.

Let the indicator $\Delta(p, t, i)$ be equal to 1, should the time period t correspond to the p-th visit to node i, 0 otherwise. Hence, instead of carrying ζ^p_{jt} from master to the subproblem, we may deduct this information from the resulting network structure of MP. Using this definition, although we can introduce the following capacity constraints and deal with the truck convoy capacity in the subproblem, our extensive computational experiments suggest to keep the fleet sizing part remains in the MP:

$$V^{cap} v^t_{0k} \geqslant \left\lceil \sum_k \sum_p (z^t_{ik} + y^t_{ki}) w_i \Delta(p, t, i) l_p \right\rceil \qquad \forall i, t. \tag{47}$$

In the following, we first present some separation algorithms for the Benders cuts and the aforementioned valid inequalities.

Separation of Benders cuts. The remaining constraints of (19)-(25) are handled in Benders Subproblem (SP). The SP is presented in the following:

(SP-MPHLPSD)

$$\min \sum_t \sum_j \sum_{k!=l} w_j c^t_{kl} x^t_{jkl} + \sum_t \sum_{k \neq MW: MW=0} F^t_{0k} v^t_{0k} \tag{48}$$

s. t.

$$\mathbf{u^1}: \quad \sum_{k \notin i,0} w_j x^t_{j0k} = \sum_p \sum_k l_p w_j \Delta(p, t, i), \qquad \forall t, j \neq 0, \tag{49}$$

$$\mathbf{u^2}: \quad \sum_{k \neq j} w_j x^t_{jkj} = \sum_p \sum_k l_p w_j \Delta(p, t, i), \qquad \forall t, j \neq 0, \tag{50}$$

$$\mathbf{u^3}: \quad \sum_{l \neq 0} w_j x^t_{jkl} - \sum_{l \neq k, k \neq 0} w_j x^t_{jlk} = 0, \qquad \forall t, j \neq 0, k \notin \{i, j\}, \tag{51}$$

$$\mathbf{u^4}: \quad x^t_{j0l} \leqslant \overline{r}^t_{0l}, \qquad \forall j, t, \tag{52}$$

$$\mathbf{u^5}: \quad x^t_{jkl} \leqslant \overline{r}^t_{kl} + \overline{y}^t_{kl}, \qquad \forall t, j, k \neq j, l \neq k, \tag{53}$$

$$\mathbf{u^6}: \quad x^t_{jkl} \leqslant \sum_j \overline{z}^t_{kj}, \qquad \forall t, j, k \neq j, l \neq k, \tag{54}$$

$$x^t_{jklt} \in (0, 1)^{|V|^3|T|}, v_{0kt} \in \mathbb{N}^{|\nu||T|}. \tag{55}$$

Let η be the continuous variable providing an underestimation of the Subpoblem objective function, the optimality Benders cut looks like the following:

$$\eta \geqslant \sum_{t, j \neq 0} u^1_{jt} \sum_p \sum_k l_p w_j z^p_{jkt} + \sum_{t, j \neq 0} u^2_{jt} \sum_p \sum_k l_p w_j z^p_{jkt} + \sum_{t, j \neq 0} u^4_{jt} r_{0lt}$$

$$+\sum_{t,j,k\neq j,k>0,l\neq k}\left(u^5_{jklt}(r_{klt}+y_{klt})+u^6_{jklt}(\sum_m z^t_{km})\right). \quad (56)$$

and the feasibility cut follows:

$$0\geqslant\sum_{t,j\neq 0}u^1_{jt}\sum_p\sum_k l_p w_j z^p_{jkt}+\sum_{t,j\neq 0}u^2_{jt}\sum_p\sum_k l_p w_j z^p_{jkt}+\sum_{t,j\neq 0}u^4_{jt}r_{0lt}$$
$$+\sum_{t,j,k\neq j,k>0,l\neq k}\left(u^5_{jklt}(r_{klt}+y_{klt})+u^6_{jklt}(\sum_m z^t_{km})\right). \quad (57)$$

However, as any infeasible network structure is separated using non-Benders cuts, we will not need to separate any Benders feasibility cut.

On the other hand, as variables y^t_{ii} do not appear in the aforementioned Benders cuts, and we would like to capture as many variables as possible from among those of MP into different cuts, the following substitution is proposed leading to cuts that also accommodate y_{ii} and y_{ij}:

For constraints (19)-(20) we have:

$$y^t_{jj}+\sum_k z^t_{jk}=\sum_p \zeta^p_{jt}\Rightarrow \quad \sum_{k\notin i,0} w_j x_{j0kt}=\sum_p\sum_k l_p w_j\Delta(p,t,i)(y^t_{jj}+\sum_k z^t_{jk}), \quad \forall j,t, \quad (58)$$
$$y^t_{jj}+\sum_k z^t_{jk}=\sum_p \zeta^p_{jt}\Rightarrow \quad \sum_{k\neq j} w_j x_{jkjt}=\sum_p\sum_k l_p w_j\Delta(p,t,i)(y^t_{jj}+\sum_k z^t_{jk}), \quad \forall j,t, \quad (59)$$
$$\sum_k(y^t_{kj}+z^t_{jk})=\sum_p \zeta^p_{jt}\Rightarrow \quad \sum_{k\notin i,0} w_j x_{j0kt}=\sum_p\sum_k l_p w_j\Delta(p,t,i)(\sum_k(y^t_{kj}+z^t_{jk})+\sum_k z^t_{jk}), \quad \forall j,t, \quad (60)$$
$$\sum_k(y^t_{kj}+z^t_{jk})=\sum_p \zeta^p_{jt}\Rightarrow \quad \sum_{k\neq j} w_j x_{jkjt}=\sum_p\sum_k l_p w_j\Delta(p,t,i)(\sum_k(y^t_{kj}+z^t_{jk})+\sum_k z^t_{jk}). \quad \forall j,t. \quad (61)$$

In this way, one can also accommodate variables y^t_{ii}, y^t_{ji}, $\forall i,j,t$ in the separated Benders cuts. From (58) - (61) the following optimality cuts are deduced (feasibility cuts follow the same style):

$$\eta\geqslant\sum_{t,j\neq 0}u^1_{jt}\sum_p\sum_k l_p w_j(y^t_{ii}+\sum_k z^t_{ik})+\sum_{t,j\neq 0}u^2_{jt}\sum_p\sum_k l_p w_j(y^t_{ii}+\sum_k z^t_{ik})$$
$$+\sum_{t,j\neq 0}u^4_{jt}r_{0lt}+\sum_{t,j,k\neq j,k>0,l\neq k}\left(u^5_{jklt}(r_{klt}+y_{klt})+u^6_{jklt}(\sum_m z^t_{km})\right) \quad (62)$$
$$\eta\geqslant\sum_{t,j\neq 0}u^1_{jt}\sum_p\sum_k l_p w_j(\sum_k(y^t_{kj}+z^t_{jk}))+\sum_{t,j\neq 0}u^2_{jt}\sum_p\sum_k l_p w_j(\sum_k(y^t_{kj}+z^t_{jk}))$$
$$+\sum_{t,j\neq 0}u^4_{jt}r_{0lt}+\sum_{t,j,k\neq j,k>0,l\neq k}\left(u^5_{jklt}(r_{klt}+y_{klt})+u^6_{jklt}(\sum_m z^t_{km})\right) \quad (63)$$

Our extensive computational experiments does suggest the use of both type of optimality cuts (56) and (62)-(63).

Multi-Cut Benders decomposition. A closer look at the subproblem and the Benders cut (56) reveals that the subproblem can be solved for every period independently and as a result one can generate up to $|T|$ cut per subproblem. Let $\eta^t \geqslant 0$ and $\eta = \sum_t \eta^t$, where $\eta^t \geqslant 0$ underestimates the cost corresponding to the period t, one can disaggregate the Benders cuts into:

$$\eta^t \geqslant \sum_{j\neq 0} u^1_{jt} \sum_p \sum_k l_p w_j (y^t_i + \sum_k z^t_{ik}) + \sum_{j\neq 0} u^2_{jt} \sum_p \sum_k l_p w_j (y^t_{ii} + \sum_k z^t_{ik})$$
$$+ \sum_{j\neq 0} u^4_{jt} r_{0lt} + \sum_{j,k\neq j,k>0,l\neq k} \left(u^5_{jklt}(r_{klt} + y_{klt}) + u^6_{jklt}(\sum_m z^t_{km}) \right). \qquad \forall t, \qquad (64)$$

Multi-cut for the feasibility cut follows the same algebra.

4.3. Cutting plane

Separation of valid inequalities (27). The separation of this class of valid inequalities is reduced to solving a Knapsack Problem (KP) with two additional constraints. The weights of items correspond to $\bar{r}^t_{ij}$. For every t we solve:

$$\begin{aligned} &\max \sum_i b_i \\ &\text{s.t.} \sum_{i,j} \bar{r}^t_{ij} s_{ij} \geqslant \sum_i b_i + \epsilon \\ &\quad s_{ij} \leqslant b_i && \forall i,j \\ &\quad s_{ij} \leqslant b_j && \forall i,j \\ &\quad b_i, s_{i,j} \in \{0,1\} && (65) \end{aligned}$$

Any feasible solution to this problem identifies a set $S^t = \{i : b_i = 1, \forall i\}$, $\forall t$ that is used to separate the corresponding valid inequalities from (27) (see Gendreau et al. (1998) for a different method).

Separation of valid inequalities (28) *and* (29). At every period t, let s be a dummy node, V^t be the set of nodes visited and $G'^t(V^t \cup \{s\}, A'^t)$ a directed graph. Establish an arc from s to every j with capacity $\bar{z}^t_{ij}$, if $\bar{z}^t_{ij} > 0$. Then, add an arc (i,j) with capacity $\bar{r}^t_{ij}$ for $\bar{r}^t_{ij} > 0$. Now, in this period t, if we pump a flow from s and destined to a node i, then i must receive a volume of flow equivalent to $\sum_j \bar{z}^t_{ij}$. A set $S \subset V^t$ where $i \in S, s \notin S$ defines a cut of $\delta^+(S)(\delta^-(S))$ if the cut capacity is less than $\sum_j \bar{z}^t_{ij}$. S will deliver valid inequalities of (28) ((29)). The complexity of separation relies on Edmonds-Karp max-flow.

Separation of valid inequalities (30) *and* (31). At every period t, let $G''^t(V^t, A''^t)$ be defined as following: V^t is the set of nodes visited and A''^t is composed of arcs (i,l) for which $\bar{r}^t_{il} + \bar{z}^t_{i'l} > 0$, arcs (i',l) for which $\bar{r}^t_{i'l} + \bar{z}^t_{il} > 0$ and all other arcs (k,l) where $\bar{r}^t_{kl} > 0$. A set $S \subset V^t$ defines a cut, if the max-flow is less than $\sum_{j\in V/S} \bar{z}^t_{ij} + \sum_{j'\in S} \bar{z}^t_{i'j'}$ where $i \in S, j \in V^t/S$. S, V^t/S where $i \in S$, $j \in V^t/S$ delivers the valid inequalities (30) and (31). One can either use Edmonds-Karp or Boykov and Kolmogorov (2001) max-flow algorithms.

Separation of valid inequalities (33). In order to identify the violated inequalities of this type, it is suggested in Martín et al. (2014) to look for a cut set S of the support graph $G'''^t(V^t, A'''^t)$ or in V^t/S. Therefore in parallel with our search for the violated inequalities of (28) (29), (30) and (31), we also use the identified cut-set, say S, to search for possible violation of (33). The complexity again depends on the max-flow algorithm of Edmonds-Karp or Boykov and Kolmogorov (2001).

Separation of valid inequalities (34). Our extensive computational experiment reveals that the efforts invested in the separation of inequalities for $|P| \geqslant 2$ do not pay off (similar conclusion has been drawn in Labbé et al. (2004)). Therefore, we would prefer to stick to the existing constraints (5).

Separation of valid inequalities (35). The separation of such constraints is similar the one in Martín et al. (2014). In period t, let $S = \{i\} \cup \{j \in V^t - \{i'\} : z^t_{ij} \geqslant z^t_{ij}\}$, for a given arc (i, i'). According to Martín et al. (2014), if the inequality is not violated for this choice of S, it is neither violated for any other set S'. The complexity is $\mathscr{O}(N^3)$.

Separation of valid inequalities (36). In period t, we establish a support graph $G^{t\ddagger} = (V^{t\ddagger}, A^{t\ddagger})$ where $V^{t\ddagger}$ is composed of all $a \in A^t$ for which $r^{t*}_a > 0$. We then enumerate all the cycles in such a graph using Tiernan's method (see Tiernan (1970)) and examine every cycle to identify the violated cuts. When enumerating all the cycles, one may also examine the violation of the inequalities (46).

Separation of valid inequalities (37). In every period t, we can rewrite (37) as $(N^{max} - 1) \sum_{e \in \delta^+(S)} r^t_a + N^{max} \sum_{i \in S} z^t_{ii} \geqslant \sum_{i \in S} \sum_{j \neq i} z^t_{ij}$ or $(N^{max}-1) \sum_{e \in \delta^+(S)} r^t_a + N^{max} \sum_{i \in S} z^t_{ii} + \sum_{i \notin S} \sum_j z^t_{ij} \geqslant \sum_{i \in V} \sum_j z^t_{ij}$. These can now be separated in the same way as the SEC in the traveling salesman problem. We establish a support graph $G^{t\S} = (V^{t\S}, A^{t\S})$ where $V^{t\S} = V^t \cup \{s, d\}$ where s and d are two dummy nodes and $A^{t\S}$ is composed of the following edges: 1) all arcs $a \in A^t$ with capacity $N^{max} - 1$ for all $r^t_a > 0$, 2) the arcs (s, i), $\forall i \in V^t$ with capacity $N^{max} z^t_{ii}$, and 3) the edges $\{i, d\}$, $\forall i \in V^t$ with capacity $\sum_j z^t_{ij}$. A set S^t where $s \in S^t, d \notin S$ represents a set that can potentially give a violated valid inequality (37). The complexity of this separation is equivalent to the complexity of an s-d min-cut algorithm called on $G^{t\S}$.

Separation of valid inequalities (38). While Fischetti et al. (1998b) and other researchers proposed several heuristic separation algorithms for such constraints, our initial computational experiments suggested using the method of Padberg and Rao (1982) for separating such valid inequalities in every period t.

Separation of valid inequalities (42). while polynomial-time algorithms exists for separation of these inequalities (see Letchford and Lodi (2002)) our initial computational experiments trades polynomial-time algorithm for the block decomposition technique. Therefore, we find combs using Concorde implementation (see (Applegate et al., 2007) and Cook's).

Separation of valid inequalities (46). In every period t, at any integer node where an inequality of form (36) is violated, every sub-path of length $N^{max} + 1$ represents such a valid inequality and is violated by 1. This separation can be done in $\mathscr{O}(n^2)$.

4.4. *Branch-and-cut algorithm*

As mentioned earlier, our emphasis in our branch-and-bound is on a balance between a fast lower bound improvements and quickly finding feasible solution. For that, we separate all possible violated cuts at the root node. If no inequality from among the non-Benders ones has been found we then generate Benders cuts.

Due to the way we dealt with ζ^p_{it} variables as indicators, the method proposed in Papadakos (2008) becomes less intuitive. We therefore, used an alternative method as follows: Once the subproblem is solved to optimality, we move over the face of optimality and choose a solution that minimized the sum of dual values corresponding to the equality constraint (i.e., $\mathbf{u^1} + \mathbf{u^2}$). This has shown to be promising.

In a gap of less than 5%, we do not separate any cut for the fractional solutions. We also opted to branch on z variables first even though one may argue that r seems more trivial candidates but our choice has been confirmed by an extensive computational experiments.

4.5. *Preprocessing*

Some practice-driven rules can be used to fix some variables in the model one of which would be the following: For every node i and $j = \operatorname{argmax} c_{ij}$, we can fix t_{ij}, r_{ji}, y_{ij} and y_{ji} to 0.

5. Computational experiments

The BENMIP framework is written in C++ and can be compiled both in Windows (Microsoft C++ compiler) and in Linux (Ubuntu). For this project, we compile BENMIP using the VC++ (Visual Studio 2017) compiler in Windows 10 and run on a personal computer with an Intel Core i7-6700K CPU, 4.0 GHz and 32 GB of RAM. CPLEX 12.8 is used as an MIP solver. The max-flow algorithms are solved using the Boost library implementation of the Edmonds-Karp algorithm for directed graphs.

Except for the case study, other instances are generate randomly for a network of size 100 nodes using the kind of aggregation proposed by Ernst and Krishnamoorthy (1996) to establish grids on the map and introduce new virtual nodes as the centers of gravity of cells in the grid with a demand equal to that of all the nodes within the grid cell. The demands are also generated randomly within the min/max range data of the case study. The random generation has been done with the aim of avoiding any obvious structure in the distance and demand matrices.

In both cases, the transportation cost accounts for the fuel consumption per 100 km for the design speed of the a trucks. We also assumed that the fleet of transporters is homogeneous one and therefore the fuel consumption pattern is the almost the same across the fleet and corresponds to 24 liters per 100 kilometers for the type of trucks being deployed. The penalty for the supply to the isolated nodes is been set to 2 (we will see in the case study section that we use the same factor over there as well).

Some CPLEX parameters are the following: `IloCplex::MIPEmphasis` has been set to `CPX_MIPEMPHASIS_BALANCED`. The custom termination criteria for CPLEX (UserAbort) is the following: we terminate CPLEX if a maximum of 30 hours (108000 seconds) of CPU time is passed and an incumbent solution has been identified. After an extensive preliminary computational experiments, we realized that for many instances, the time limit is reached while more than 90% of the time (currently marked by 'OptimTol') has been spent on closing the last 0.5% gap (reducing the gap from 0.5% to zero to prove the optimality). We therefore decided to modify the criteria by adding an optimality tolerance of 0.5%. The results reported here use this combined termination criteria.

$|\mathscr{T}|$ is chosen from the set $\{3, 5, 7, 9\}$ due to the intractability of problem. As long as Benders decomposition is concerned, we only separate cuts upon finding an optimal integer proposal being found by CPLEX. Other cuts are separated at root node and later on every 10 nodes (fractional or integer) and any integer node. The tolerance $\epsilon = 10^{-4}$ was set as the threshold of violation of separated valid cuts. Our extensive computational experiments confirmed that the approach itself is sufficiently efficient and that further efforts to generate sharper cuts from the SP only add extra overheads without making any significant improvements. Moreover, N^{min} and N^{max} are set to 3 and 5, respectively, as it is less realistic to visit more than 4 locations (other that the DC itself) per day and is less likely to deploy a fleet for only 1 or two nodes on a route.

The demand pattern (user's behavior) for different length of planning horizon is reported in Table 4. Those rows for 6 and 7 days are obtained through simulation as we only had access to 1-5 days behavior pattern in our case study. The interpretations follows: assuming 3 deliveries, 38% of the refugees collect their demands on the first day, 24% collect on the second day and 38% on the last day. Our problem considers a planning horizon of at least 5 days long and minimum three deliveries.

We deactivated several internal cut generation procedures of CPLEX. These include FlowCovers, FlowPaths, MIRCuts, FracCuts, LiftProjCuts, Cliques, Covers, FlowCovers, GUBCovers, FracCuts, MIRCuts, DisjCuts, ZeroHalfCuts and MCFCuts. We turned off all the presolved phases to avoid premature convergence in Benders.

In Table 5, we report our computational experiments. The first column represents the instance name in the format 'N, T, q', where N and T refer to the number of nodes and number of periods, and q represents

# days	# Fraction of demand per day
1	100%
2	50% and 50%
3	38%, 24% and 38%
4	31%, 21%, 14% and 34%
5	23%, 18%, 18%, 20% and 21%
6	21%, 14%, 13%, 14%, 18% and 20%
7	19%, 13%, 12%, 13%, 13%, 12% and 18%

Table 4: The demand patter for different planning horizon lengths (rounded to integer values).

the required number of visits, respectively. The column 'CPLEX' represents the results of direct application of CPLEX to solve the instance with a time limit of 30 hours. In the column 'TimeIP', we report the elapsed time for solving the problem at hand before CPLEX terminated. The column 'Nnodes' represents the number of processed nodes. The columns 'Gap' and 'MIPStatus' stand for the integrality gap and the CPLEX status, respectively. The best known objective value (proven optimal when gap equals 0) is presented in the column 'bestObj'. In the column 'NUserCuts', we report the number of user cuts added in the course of the branch, Benders and cut approach.

As depicted in Table 5, using our branch, cut and Benders (BCB), we are able to solve to optimality, instance with up to 45 nodes, 9 periods and 5 visits in a very reasonable amount of time. From 57 nodes and 9 periods, up to 75 nodes and 9 periods, the optimal tolerance of 0.5% is met and for the remaining instances, bCB process failed. At the same time, given the time limit of 108,000 seconds, CPLEX can only solve 1 instance to optimality, six instances to a gap less than 1 percent within the time limit and all other cases failed even to produce any feasible solution.

While number of separated user cuts, including Benders and non-Benders cuts –even for larger size instance– remains reasonable for smaller size instances (smaller $N \times T \times q$), this becomes excessively higher for larger size instances. The number of processed nodes is reasonable and the number of separated cuts is increasing as $N \times T$ increases. This makes the LP increasingly difficult to resolve.

We tried to let the algorithm continue even further beyond this time, but the additional time spent did not pay off with any better solution quality. The numbers of separated valid inequalities and Benders cuts are rather reasonable and increase as the size of the instance increases.

In general, we are able to solve relatively large size instances to less than 0.5% gap and in reasonable time.

N, T, q	CPLEX	TimeMIP	Nnodes	Gap	IPStatus	bestObj	NUserCuts
15, 5, 3	TL ($\leqslant$ 1%)	7.994	180	0.00	**Optimal**	84080.337	451
15, 5, 5	TL ($\leqslant$ 1%)	12.457	3600	0.00	**Optimal**	111658.915	1249
20, 5, 3	TL ($\leqslant$ 1%)	10.673	4560	0.00	**Optimal**	37523.286	521
20, 7, 5	TL ($\leqslant$ 1%)	19.997	100	0.00	**Optimal**	76397.008	61
25, 5, 3	TL ($\leqslant$ 1%)	21.287	1950	0.00	**Optimal**	84241.285	198
25, 7, 5	TL ($\leqslant$ 1%)	27.476	12000	0.00	**Optimal**	125209.490	1049
25, 9, 7	Failed	27.559	6300	0.00	**Optimal**	113419.935	8247
30, 5, 3	Failed	42.306	10350	0.00	**Optimal**	43762.929	78
30, 7, 5	Failed	42.766	21750	0.00	**Optimal**	162934.996	1687
30, 9, 5	Failed	52.454	14280	0.00	**Optimal**	228946.761	10834
30, 9, 7	Failed	51.198	23220	0.00	**Optimal**	189169.091	24173
35, 5, 3	Failed	66.435	3675	0.00	**Optimal**	244242.668	264
35, 7, 5	Failed	85.371	8750	0.00	**Optimal**	62413.852	2243

N, T, q	CPLEX	TimeMIP	Nnodes	Gap	IPStatus	bestObj	NUserCuts
35, 9, 5	Failed	127.431	34545	0.00	**Optimal**	260694.174	10386
35, 9, 7	Failed	158.242	16380	0.00	**Optimal**	197920.988	49094
40, 5, 3	Failed	114.198	17040	0.00	**Optimal**	199238.713	380
40, 7, 5	Failed	104.832	1400	0.00	**Optimal**	221238.622	2864
40, 9, 7	Failed	146.383	21840	0.00	**Optimal**	99243.650	9397
40, 11, 9	Failed	201.236	57240	0.00	**Optimal**	113640.182	48738
45, 5, 3	Failed	151.405	23490	0.00	**Optimal**	133406.385	157
45, 7, 5	Failed	274.801	11475	0.00	**Optimal**	141642.879	944
45, 9, 5	Failed	226.616	18270	0.00	**Optimal**	248718.573	2069
45, 9, 7	Failed	276.357	81405	0.22	OptimTol	324613.570	10625
50, 5, 3	Failed	201.975	4800	0.40	OptimTol	319585.176	569
50, 7, 5	Failed	247.583	34500	0.33	OptimTol	317347.706	775
50, 9, 5	Failed	460.927	48300	0.24	OptimTol	188794.955	11819
50, 9, 7	Failed	643.601	92700	0.28	OptimTol	161583.087	15033
55, 5, 3	Failed	437.192	32670	0.21	OptimTol	240662.953	682
55, 7, 5	Failed	528.991	70125	0.21	OptimTol	272452.559	2326
55, 9, 5	Failed	546.226	32725	0.19	OptimTol	209048.260	8273
55, 9, 7	Failed	1083.283	90090	0.23	OptimTol	290939.990	7160
60, 5, 3	Failed	642.872	43920	0.45	OptimTol	97707.369	189
60, 7, 5	Failed	791.262	39900	0.22	OptimTol	285251.915	3169
60, 9, 5	Failed	1088.335	40740	0.55	OptimTol	205488.432	11111
60, 9, 7	Failed	1689.299	26460	0.23	OptimTol	128652.405	40725
65, 5, 3	Failed	1185.403	62400	0.21	OptimTol	565340.164	552
65, 7, 5	Failed	1559.322	39650	0.44	OptimTol	226975.650	2030
65, 9, 5	Failed	2649.085	111930	0.17	OptimTol	180398.160	7695
65, 9, 7	Failed	2460.353	90090	0.20	OptimTol	124551.237	34168
70, 5, 3	Failed	1500.802	41370	0.31	OptimTol	291988.072	521
70, 7, 5	Failed	2024.516	3850	0.24	OptimTol	356038.449	1239
70, 9, 5	Failed	3229.132	79380	0.18	OptimTol	212206.747	11906
70, 9, 7	Failed	4981.246	115290	0.18	OptimTol	316362.074	50449
75, 5, 3	Failed	4710.788	29925	0.18	OptimTol	374935.459	186
75, 7, 5	Failed	4032.050	65250	0.15	OptimTol	209856.169	2870
75, 9, 5	Failed	6931.777	182700	0.18	OptimTol	545603.181	5818
75, 9, 7	Failed	8153.850	122175	0.17	OptimTol	219288.648	15106
80, 5, 3	Failed	5456.438	62640	4.64	Failed	382013.703	277
80, 7, 5	Failed	7715.971	116400	1.01	Failed	643062.686	2374
80, 9, 5	Failed	10511.407	100800	1.04	Failed	340879.407	8450
80, 9, 7	Failed	8042.656	25200	1.20	Failed	373209.248	1299
85, 5, 3	Failed	10046.642	99705	1.28	Failed	417598.357	205
85, 7, 5	Failed	9325.384	15300	23.47	Failed	751982.377	3078
85, 9, 5	Failed	17163.296	111860	1.62	Failed	373611.634	8838
85, 9, 7	Failed	12968.537	21420	1.27	Failed	560609.787	30600
90, 5, 3	Failed	20909.273	30510	1.12	Failed	250690.119	519
90, 7, 5	Failed	17782.017	202050	1.93	Failed	369989.614	179
90, 9, 5	Failed	17509.658	10080	6.71	Failed	651969.202	2819
90, 9, 7	Failed	36566.741	281880	1.61	Failed	226597.933	19589
95, 5, 3	Failed	24118.205	9405	2.80	Failed	407999.198	155
95, 7, 5	Failed	22318.252	2375	1.78	Failed	314163.261	2627
95, 9, 5	Failed	34497.094	93100	7.83	Failed	171951.665	129
95, 9, 7	Failed	42681.357	213750	4.68	Failed	891420.488	46152

N, T, q	CPLEX	TimeMIP	Nnodes	Gap	IPStatus	bestObj	NUserCuts
100, 5, 3	Failed	37314.855	13500	4.36	Failed	288486.917	474
100, 7, 5	Failed	59853.853	80500	1.52	Failed	211461.544	617
100, 9, 5	Failed	70290.977	319200	6.44	Failed	433505.924	4487
100, 9, 7	Failed	57030.752	317700	2.88	Failed	547141.994	47010

Table 5: Branch, Benders and Cut for the small to mid-size instances.

District	Name	LAT	LON	Population
1	Akkar	34.55495	36.19510	143,634
2	El Batroun	34.25484	35.65896	20,260
3	Bcharre	34.25043	36.01056	3,945
4	El Koura	34.33810	35.80584	23,097
5	El Minieh-Dennie	34.41000	35.96000	81,668
6	Tripoli	34.44536	35.82251	76,018
6	Zgharta	34.39630	35.89580	17,775
7	Rachaya	33.52851	35.89803	14,393
8	West Bekaa	33.42000	35.85000	91,054
10	Zahle	33.84756	35.90271	241,235
11	Baalbek	34.00965	36.21171	172,115
12	El Hermel	34.39620	36.38704	8,652
13	Beirut	33.89592	35.47843	37,271
14	Aley	33.80780	35.60502	86,069
15	Baabda	33.83392	35.54435	128,878
16	Chouf	33.89016	35.49363	73,270
17	El Meten	33.89190	35.56300	75,314
18	Jbeil	33.89937	35.49675	9,347
19	Kesrwane	34.01323	35.80083	23,641
20	Jezzine	33.54393	35.58439	4,379
21	Saida	33.56142	35.37661	62,557
22	Sour	33.27212	35.19640	39,573
23	Bent Jbeil	33.11894	35.43411	11,329
24	Hasbaya	33.39801	35.68202	8,020
25	Marjaayoun	33.24016	35.44718	10,371
26	El Nabatieh	33.38113	35.48260	36,136

Table 6: Distribution of refugees in 26 Lebanese districts.

5.1. Case study

The Lebanese case study is composed of 26 nodes as the centers of districts and one main warehouse (MW) —27 nodes in total. The planning horizon is a week (five working days, $T = 5$) and every node is served in three different days, i.e. $q = 3$. In Table 6, the first column represents name, followed by lat/lon coordinates and the last column representing population size. In all figures, we use '0' (zero in the darker box) to refer to MW and all others numbers follow the order in this table. The distance table is calculated from the road transport distance (queried using Google API). As before, we assumed a homogeneous fleet of trucks and therefore the fuel consumption is approximately the same for all the trucks and equivalent to 24 litter per 100 KM. We further assumed that penalty factor for not serving a node directly is equal to 2.0

—twice more expensive per unit of volume per kilometer compared to the normal service.

The reported solution is an optimal solution of the case-study with 26 nodes.

The DCs and isolated node for every day are reported in Table 7 and Table 8:

Day	DCs
Monday	6, **9**, **10**, **15**,
Tuesday	5, **9**, **10**, **15**, **16**, **26**,
Wednesday	**9**, **10**,
Thursday	**7**, **16**, 25,
Friday	**7**, **15**, **16**, and **26**

Table 7: The DCs in the optimal solution of Lebanese case study.

Day	Isolated nodes
Monday	[**3**](10), [20,21] (15), [**23**](9),
Tuesday	[1,**12**](5), [**3**](10), [**2**](16), [**22**,25](26),
Wednesday	[**23**](9), [**3**](10),
Thursday	[**2**,**12**](7), [**22**](25),
Friday	[**12**](7), [20](15), [**2**](16), and [**22**](26)

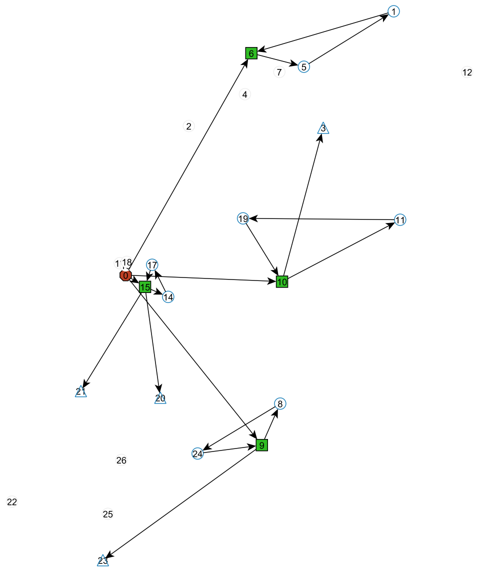
Table 8: The isolated nodes in the optimal solution of Lebanese case study.

At least two and at most 6 DCs are planned for every day. The nodes 10 (Zahle), 15 (Baabda) and 16 (Chouf) are appearing as DCs in all q times. These nodes are among the top 10 nodes with respect to the demand size. The node 10 (Zahle) is the node with highest demand while 16 (Chouf) and 15 (Baada) are among the nodes with high demand —even though not the highest. One observes in Figure 5 that, geographically speaking, these nodes are located at the interior par of the convex hull of the selected nodes of every service day. A combination of higher demand and median-like property encourage them to be the DCs of the days.

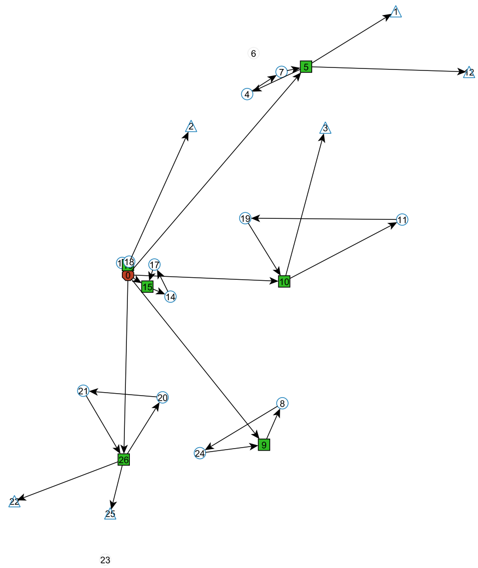
As per Table 8, between 2 and 6 nodes are considered as isolated nodes in daily services. The nodes 2 (El Batroun), 3 (Bcharre), 12 (El Hermel) and 23 (Bent Jbeil) are among the nodes with lowest demands (among the 10 lowest in the table) and are designated as isolated in all q times visit. Moreover, as can be observed in Figure 5, geographically speaking, these nodes are located at the perimeter (say are the outliers) of the selected nodes of every service day. lower volume of demand and the geographical position of nodes is an important factor in designating them as isolated nodes in an optimal solution.

6. Summary and Conclusion

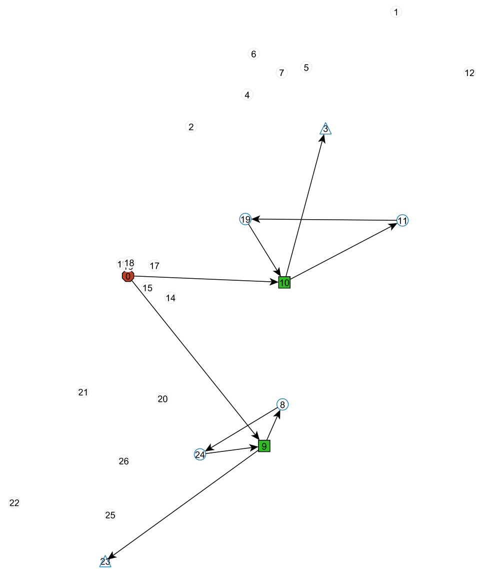
We proposed the first *Multiperiod Hub Location Problem with Serial Demand (MPHLPSD)* accommodating user's demand pattern, and formulated it as a MILP. We showed that the problem of distributing humanitarian aids could be formulated as such a problem. We further showed that a Benders formulation of problem can help getting rid of some unnecessary variables and benefit from logical relationships among variables. Several classes of valid inequalities and efficient separation routines have been identified, which resulted in an efficient branch-and-cut and Benders approach capable of solving relatively lager size instances, which are far too challenging for general-purpose solvers. We also showed that while a general-purpose solver is



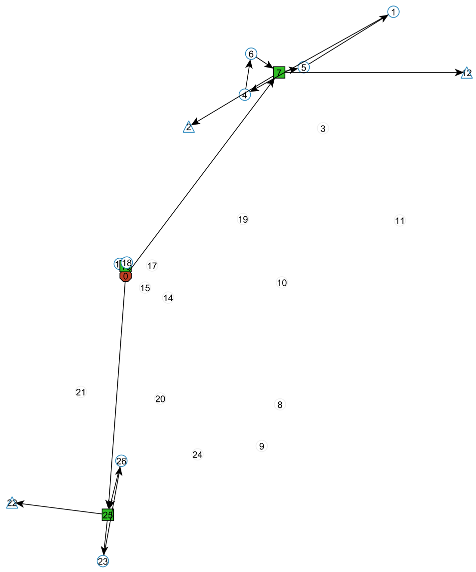
(a) Monday.



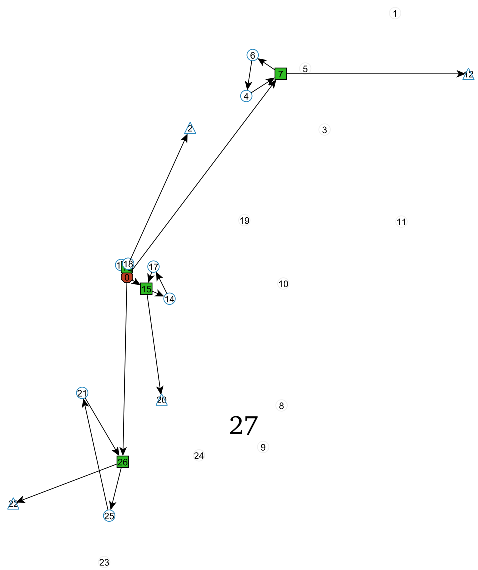
(b) Tuesday.



(c) Wednesday.



(d) Thursday.



(e) Friday.

Figure 5: An optimal solution of network in our case study.

often unable to even find a feasible solution for instances of this problem, such a solution can be generated easily and be used to warm start our branch, cut and Benders method. On the real-life case study we reported an optimal solution and it revealed some interesting observations.

Further research directions include polyhedral analysis. Specially, identifying new tightening valid inequalities could count as an objective. Also, improving the performance of separation routines is of importance. Moreover, the hybridization of metaheuristic and exact methods, which can deliver solutions with known quality, deserves a great attention.

7. Acknowledgments

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