THE COMPLEXITY OF HARD GRAPH PROBLEMS THIRTY YEARS LATER

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GRAPH CLASS	MEMBER		INDSET		CLIQUE		CLIPAR		CHRNUM		CHRIND		HAMCIR		DOMSET		MAXCUT		STTREE		GRAPHISO	
Trees/Forests	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[GJ]	P	[T]	P	[GJ]
Almost Trees (k)	P		P	[OG]	P	[T]	P?		P?		P?		P?		P	[OG]	P?		P?		P?	
Partial k-trees	P	[OG]	P	[OG]	P	[T]	**P**	**[S]**	P	[OG]	**P**	**[S]**	P	[OG]	P	[OG]	**P**	**[S]**	**P**	**[S]**	**P**	**[S]**
Bandwidth-k	P	[OG]	P	[OG]	P	[T]	P?		P	[OG]	P?		P?		P	[OG]	P	[OG]	P?		P	[OG]
Degree-k	P	[T]	N	[GJ]	P	[T]	N	[GJ]	N	[GJ]	N	[OG]	N	[GJ]	N	[GJ]	N	[GJ]	N	[GJ]	P	[OG]
Planar	P	[GJ]	N	[GJ]	P	[T]	N	[OG]	N	[GJ]	O		N	[GJ]	N	[GJ]	P	[GJ]	N	[OG]	P	[GJ]
Series Parallel	P	[OG]	P	[OG]	P	[T]	**P**	**[S]**	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[GJ]	P	[OG]	P	[GJ]
Outerplanar	P		P	[OG]	P	[T]	P	[OG]	P	[OG]	P	[OG]	P	[T]	P	[OG]	P	[GJ]	P	[OG]	P	[GJ]
Halin	P		P	[OG]	P	[T]	P	[OG]	P	[OG]	P	[OG]	P	[T]	P	[OG]	P	[GJ]	**P**	**[S]**	P	[GJ]
k-Outerplanar	P		P	[OG]	P	[T]	P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]	P	[GJ]	P?		P	[GJ]
Grid	P		P	[GJ]	P	[T]	P	[T]	P	[T]	P	[GJ]	N	[OG]	N	[OG]	P	[T]	N	[OG]	P	[GJ]
$K_{3,3}$-Free	P	[OG]	N	[GJ]	P	[T]	N	[GJ]	N	[GJ]	O?		N	[GJ]	N	[GJ]	P	[OG]	N	[GJ]	**I**	**[S]**
Thickness-k	N	[OG]	P	[GJ]	P	[T]	N	[GJ]	N	[GJ]	N	[OG]	N	[GJ]	N	[GJ]	N	[OG]	N	[GJ]	O?	
Genus-k	P	[OG]	P	[GJ]	P	[T]	N	[GJ]	N	[GJ]	O?		N	[GJ]	N	[GJ]	O?		N	[GJ]	P	[OG]
Perfect	**P**	**[S]**	P	[OG]	P	[OG]	P	[OG]	P	[OG]	**N**	**[S]**	N	[OG]	N	[OG]	**N**	**[S]**	N	[GJ]	I	[GJ]
Chordal	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		N	[OG]	N	[OG]	**N**	**[S]**	N	[OG]	I	[GJ]
Split	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		N	[OG]	N	[OG]	**N**	**[S]**	N	[OG]	I	[OG]
Strongly Chordal	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		**N**	**[S]**	P	[OG]	**N**	**[S]**	P	[OG]	**I**	**[S]**
Comparability	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	**N**	**[S]**	N	[OG]	N	[OG]	**N**	**[S]**	N	[GJ]	I	[GJ]
Bipartite	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	N	[OG]	N	[OG]	P	[T]	N	[GJ]	I	[GJ]
Permutation	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		**P**	**[S]**	P	[OG]	O?		P	[OG]	P	[OG]
Cographs	P	[T]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]	**P**	**[S]**	P	[OG]	P	[OG]
Undirected Path	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		**N**	**[S]**	N	[OG]	**N**	**[S]**	O?		I	[GJ]
Directed Path	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		**N**	**[S]**	P	[OG]	O?		P	[OG]	O?	
Interval	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]
Circular Arc	P	[OG]	P	[OG]	P	[OG]	P	[OG]	N	[OG]	O?		**P**	**[S]**	P	[OG]	O?		P	[OG]	O?	
Circle	P	[OG]	P	[GJ]	P	[OG]	**N**	**[S]**	N	[OG]	O?		P	[OG]	**N**	**[S]**	**N**	**[S]**	P	[OG]	O?	
Proper Circ. Arc	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]	O?		P	[OG]	**P**	**[S]**
Edge (or Line)	P	[OG]	P	[GJ]	P	[T]	N	[GJ]	N	[OG]	**N**	**[S]**	N	[OG]	N	[GJ]	**P**	**[S]**	N	[OG]	I	[OG]
Claw-Free	P	[T]	P	[OG]	**N**	**[S]**	N	[GJ]	N	[OG]	**N**	**[S]**	N	[OG]	N	[GJ]	**N**	**[S]**	N	[OG]	I	[OG]

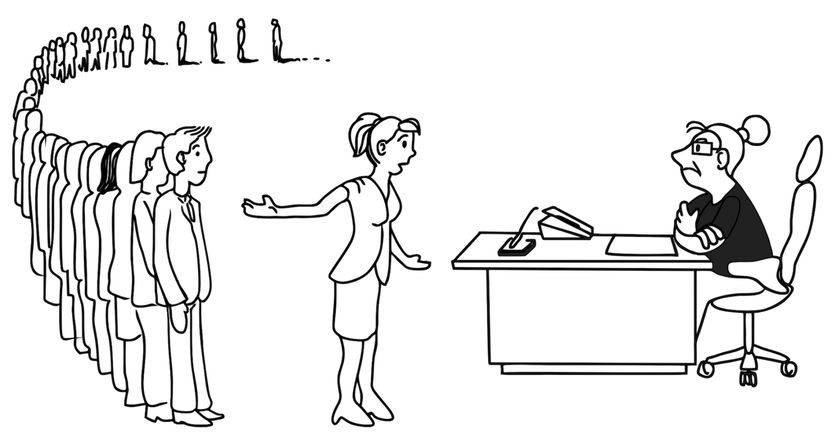
The updated table from 1985 to 2018.

There are 23 new references, classifying 33 former open problems.

There are 33 new entries, all in bold, please refer to the survey paper [S].

We keep the abbreviations used in
The NP-Completeness Column: An Ongoing Guide [OG]:

- P = Polynomial-time solvable
- P? = Appears to be polynomial-time by standard techniques
- N = NP-complete
- I = Open, but equivalent in complexity to general GRAPH ISOMORPHISM
- O? = Apparently open, but possibly easy to resolve
- O = Open, and may well be hard
- [T] = restriction trivializes the problem



"I can't find an efficient algorithm, but neither can all these famous people."

[GJ] M.R. Garey, D.S. Johnson, Computers and Intractability, A Guide to the Theory of NP-completeness, WH Freeman, 1979.

[OG] D.S. Johnson, Graph restrictions and their effect, J. Algorithms 6 (1985) 434–451.

[S] C.M.H. de Figueiredo, Complexity-separating graph classes for vertex, edge and total-colouring, Discrete Applied Math. (2019).