The complexity of hard graph problems forty years later

Celina Miraglia Herrera de Figueiredo

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Intratabilidade e Otimização

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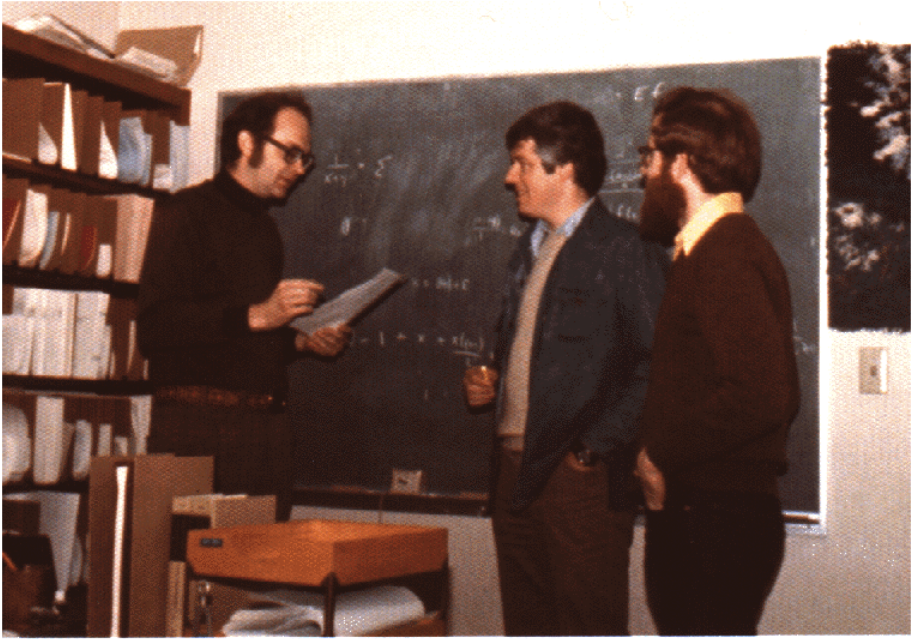
encontro de teoria da computação
congresso da sbc · porto alegre · 2016

Bio – AT&T Labs – Down the Hall

- M.R. Garey, R. L. Graham, D.S. Johnson, and D.E. Knuth
  Complexity results for bandwidth minimization
  SIAM J. Appl. Math. 34 (1978), 477–495

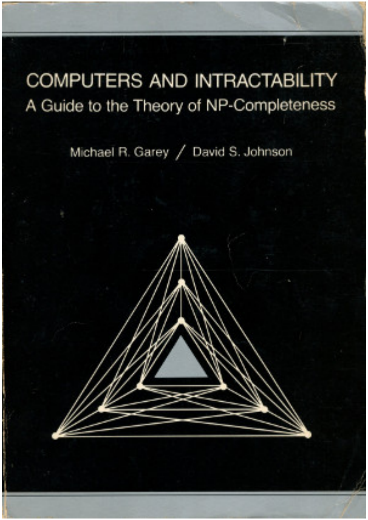
- M.R. Garey, D.S. Johnson, and R.E. Tarjan
  The planar Hamiltonian circuit problem is NP-complete
  SIAM J. Computing 5 (1976), 704–714

Knuth – Garey – Johnson



Tarjan – Garey – Johnson

The Guide – Computers and Intractability



"Despite that 23 years have passed since its publication, I consider Garey and Johnson the single most important book on my office bookshelf. Every computer scientist should have this book on their shelves as well. NP-completeness is the single most important concept to come out of theoretical computer science and no book covers it as well as Garey and Johnson."

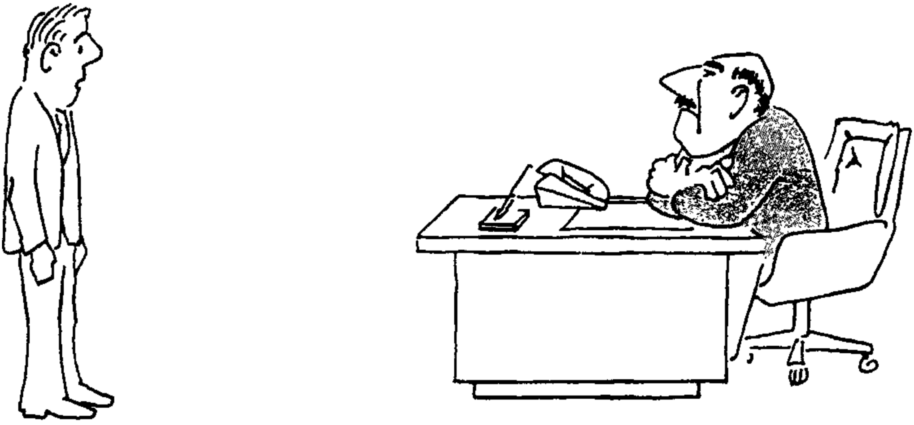
Lance Fortnow, "Great Books: Computers and Intractability: A Guide to the Theory of NP-Completeness"

NP-completo: simboliza o abismo da intratabilidade inerente para resolver problemas maiores e mais complexos

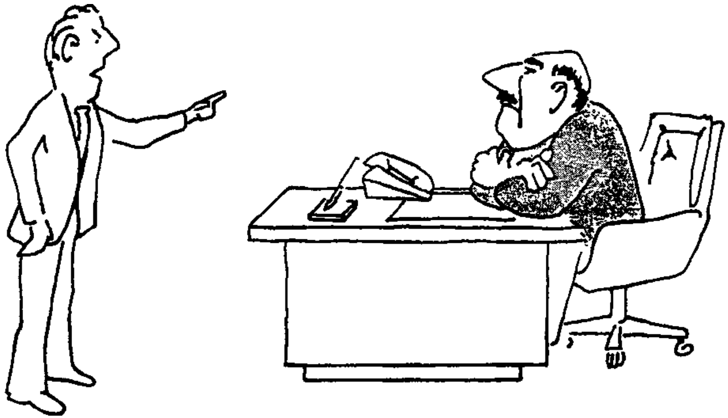
Variedade ampla de problemas frequentes: matemática, computação, pesquisa operacional

- Capítulos 1–5: teoria básica
- Capítulos 6–7: aproximação, hierarquia de classes de complexidade
- Apêndice: metade do livro! Lista bem organizada de problemas

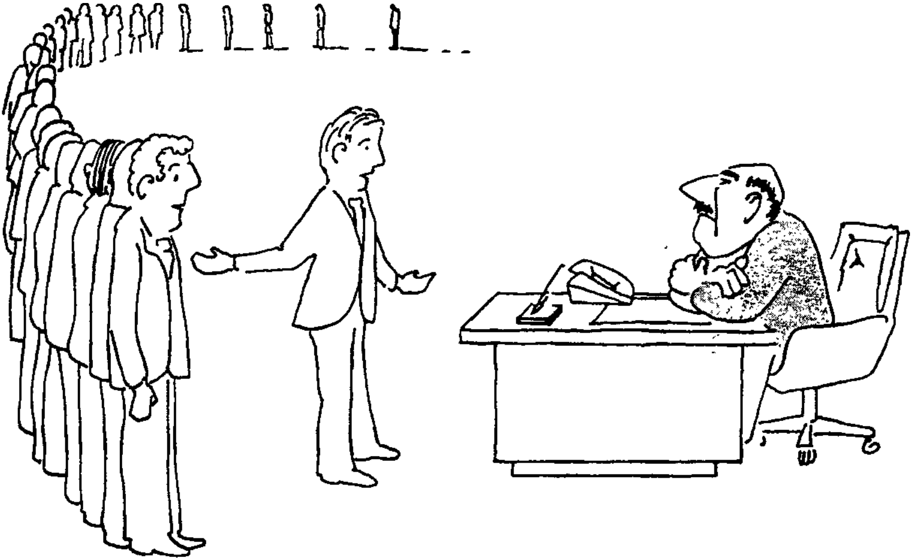
"Bandersnatches are the subject of a difficult algorithm design project for an apparently NP-complete problem."



“I can’t find an efficient algorithm, I guess I’m just too dumb.”



"I can't find an efficient algorithm, because no such algorithm is possible!"



"I can't find an efficient algorithm, but neither can all these famous people."

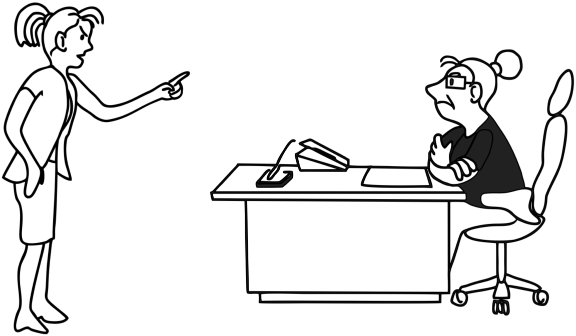
The Lost Cartoon

WE MAY NOT BE ABLE TO SOLVE IT...
BUT WE SURE CAN GET CLOSE!

The Updated Cartoons

"I can't find an efficient algorithm, I guess I'm just too dumb."

The Updated Cartoons



"I can't find an efficient algorithm, because no such algorithm is possible!"

Stefan Szeide · www.ac.tuwien.ac.at/people/szeider/cartoon/

The Updated Cartoons

“I can’t find an efficient algorithm, but neither can all these famous people.”

12 problemas em aberto em 1979

- Graph isomorphism
- Subgraph homeomorphism (for a fixed graph H)
- Graph genus
- Chordal graph completion
- Chromatic index
- Spanning tree parity problem
- Partial order dimension
- Precedence constrained 3-processor scheduling
- Linear programming
- Total unimodularity
- Composite number
- Minimum length triangulation

Ongoing Guide – Os 12 problemas atualizados em 2005

Problem Name	Source	Status	Covered in
GRAPH ISOMORPHISM	[G&J]	Open	–
SUBGRAPH HOMEOMORPHISM (FOR A FIXED GRAPH H)	[G&J]	P	[Col 19, 1987]
GRAPH GENUS	[G&J]	NPC	[Col 21, 1988]
CHORDAL GRAPH COMPLETION	[G&J]	NPC	[Col 1, 1981]
CHROMATIC INDEX	[G&J]	NPC	[Col 1, 1981]
PARTIAL ORDER DIMENSION	[G&J]	NPC	[Col 1, 1981]
PRECEDENCE CONSTRAINED 3-PROCESSOR SCHEDULING	[G&J]	Open	–
LINEAR PROGRAMMING	[G&J]	P	[Col 1, 1981]
TOTAL UNIMODULARITY	[G&J]	P	[Col 1, 1981]
SPANNING TREE PARITY PROBLEM	[G&J]	P	[Col 1, 1981]
COMPOSITE NUMBER	[G&J]	P	This Column
MINIMUM LENGTH TRIANGULATION	[G&J]	Open	–
IMPERFECT GRAPH	[Col 1, 1981]	P	This Column
GRAPH THICKNESS	[Col 2, 1982]	NPC	[Col 5, 1982]
EVEN COVER (MINIMUM WEIGHT CODEWORD)	[Col 3, 1982]	NPC	This Column
"UNRESTRICTED" TWO-LAYER CHANNEL ROUTING	[Col 5, 1982]	Open	–
GRACEFUL GRAPH	[Col 6, 1983]	Open	–
ANDREEV'S PROBLEM	[Col 17, 1986]	Open	–
SHORTEST VECTOR IN A LATTICE	[Col 18, 1986]	"NPC"	This Column

Ongoing Guide – Graph Restrictions and Their Effect

Graph Class	Member	IndSet	Clique	CliPar	ChrNum	ChrInd	HamCir	DomSet	MaxCut	StTree	GraIso
Trees/Forests	P [T]	P [GJ]	P [T]	P [GJ]	P [T]	P [GJ]	P [T]	P [GJ]	P [GJ]	P [T]	P [GJ]
Almost Trees (k)	P	P [24]	P [T]	P?	P?	P?	P?	P [45]	P?	P?	P?
Partial k-Trees	P [2]	P [1]	P [T]	P?	P [1]	O?	P [3]	P [3]	P?	P?	O?
Bandwidth-k	P [68]	P [64]	P [T]	P?	P [64]	P?	P?	P [64]	P [64]	P?	P [58]
Degree-k	P [T]	N [GJ]	P [T]	N [GJ]	N [GJ]	N [49]	N [GJ]	N [GJ]	N [GJ]	N [GJ]	P [58]
Planar	P [GJ]	N [GJ]	P [T]	N [10]	N [GJ]	O	N [GJ]	N [GJ]	P [GJ]	N [35]	P [GJ]
Series Parallel	P [79]	P [75]	P [T]	P?	P [74]	P [74]	P [74]	P [54]	P [GJ]	P [82]	P [GJ]
Outerplanar	P	P [6]	P [T]	P [6]	P [67]	P [67]	P [T]	P [6]	P [GJ]	P [81]	P [GJ]
Halin	P	P [6]	P [T]	P [6]	P [74]	P [74]	P [T]	P [6]	P [GJ]	P?	P [GJ]
k-Outerplanar	P	P [6]	P [T]	P [6]	P [6]	O?	P [6]	P [6]	P [GJ]	P?	P [GJ]
Grid	P	P [GJ]	P [T]	P [GJ]	P [T]	P [GJ]	N [51]	N [55]	P [T]	N [35]	P [GJ]
$K_{3,3}$-Free	P [4]	N [GJ]	P [T]	N [10]	N [GJ]	O?	N [GJ]	N [GJ]	P [5]	N [GJ]	O?
Thickness-k	N [60]	N [GJ]	P [T]	N [10]	N [GJ]	N [49]	N [GJ]	N [GJ]	N [7]	N [GJ]	O?
Genus-k	P [34]	N [GJ]	P [T]	N [10]	N [GJ]	O?	N [GJ]	N [GJ]	O?	N [GJ]	P [61]
Perfect	O!	P [42]	P [42]	P [42]	P [42]	O?	N [1]	N [14]	O?	N [GJ]	I [GJ]
Chordal	P [76]	P [40]	P [40]	P [40]	P [40]	O?	N [22]	N [14]	O?	N [83]	I [GJ]
Split	P [40]	P [40]	P [40]	P [40]	P [40]	O?	N [22]	N [19]	O?	N [83]	I [15]
Strongly Chordal	P [31]	P [40]	P [40]	P [40]	P [40]	O?	O?	P [32]	O?	P [83]	O?
Comparability	P [40]	P [40]	P [40]	P [40]	P [40]	O?	N [1]	N [28]	O?	N [GJ]	I [GJ]
Bipartite	P [T]	P [GJ]	P [T]	P [GJ]	P [T]	P [GJ]	N [1]	N [28]	P [T]	N [GJ]	I [GJ]
Permutation	P [40]	P [40]	P [40]	P [40]	P [40]	O?	O	P [33]	O?	P [23]	P [21]
Cographs	P [T]	P [40]	P [40]	P [40]	P [40]	O?	P [25]	P [33]	O?	P [23]	P [25]
Undirected Path	P [39]	P [40]	P [40]	P [40]	P [40]	O?	O?	N [16]	O?	O?	I [GJ]
Directed Path	P [38]	P [40]	P [40]	P [40]	P [40]	O?	O?	P [16]	O?	P [83]	O?
Interval	P [17]	P [44]	P [44]	P [44]	P [44]	O?	P [53]	P [16]	O?	P [83]	P [57]
Circular Arc	P [78]	P [44]	P [50]	P [44]	N [36]	O?	O?	P [13]	O?	P [83]	O?
Circle	P [71]	P [GJ]	P [50]	O?	N [36]	O?	P [12]	O?	O?	P [70]	O?
Proper Circ. Arc	P [77]	P [44]	P [50]	P [44]	P [66]	O?	P [12]	P [13]	O?	P [83]	O?
Edge (or Line)	P [47]	P [GJ]	P [T]	N [GJ]	N [49]	O?	N [11]	N [GJ]	O?	N [70]	I [15]
Claw-Free	P [T]	P [63]	O?	N [GJ]	N [49]	O?	N [11]	N [GJ]	O?	N [70]	I [15]

Complexity-separating graph classes for vertex, edge and total coloring

Celina de Figueiredo

COPPE UFRJ

LAGOS 2017

Overview

Classification into P or NP-complete of challenging problems in graph theory

Full dichotomy: class of problems where each problem is classified into P or NP-complete

Coloring problems: vertex, edge, total

NP-completeness ongoing guide

Identification of an interesting problem, of an interesting graph class

Categorization of the problem according to its complexity status

Problems and complexity-separating graph classes

Graph classes and complexity-separating problems

Johnson's NP-completeness column 1985

Spinrad's book 2003

Ongoing Guide – graph restrictions and their effect

GRAPH CLASS	MEMBER		INDSET		CLIQUE		CLIPAR		CHRNUM		CHRIND		HAMCIR		DOMSET		MAXCUT		STTREE		GRAISO	
Trees/Forests	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[GJ]	P	[T]	P	[GJ]
Almost Trees (k)	P		P	[24]	P	[T]	P?		P?		P?		P?		P	[45]	P?		P?		P?	
Partial k-Trees	P	[2]	P	[1]	P	[T]	P?		P	[1]	O?		P	[3]	P	[3]	P?		P?		O?	
Bandwidth-k	P	[68]	P	[64]	P	[T]	P?		P	[64]	P?		P?		P	[64]	P	[64]	P?		P	[58]
Degree-k	P	[T]	N	[GJ]	P	[T]	N	[GJ]	N	[GJ]	N	[49]	N	[GJ]	N	[GJ]	N	[GJ]	N	[GJ]	P	[58]
Planar	P	[GJ]	N	[GJ]	P	[T]	N	[10]	N	[GJ]	O		N	[GJ]	N	[GJ]	P	[GJ]	N	[35]	P	[GJ]
Series Parallel	P	[79]	P	[75]	P	[T]	P?		P	[74]	P	[74]	P	[74]	P	[54]	P	[GJ]	P	[82]	P	[GJ]
Outerplanar	P		P	[6]	P	[T]	P	[6]	P	[67]	P	[67]	P	[T]	P	[6]	P	[GJ]	P	[81]	P	[GJ]
Halin	P		P	[6]	P	[T]	P	[6]	P	[74]	P	[74]	P	[T]	P	[6]	P	[GJ]	P?		P	[GJ]
k-Outerplanar	P		P	[6]	P	[T]	P	[6]	P	[6]	O?		P	[6]	P	[6]	P	[GJ]	P?		P	[GJ]
Grid	P		P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	N	[51]	N	[55]	P	[T]	N	[35]	P	[GJ]
$K_{3,3}$-Free	P	[4]	N	[GJ]	P	[T]	N	[10]	N	[GJ]	O?		N	[GJ]	N	[GJ]	P	[5]	N	[GJ]	O?	
Thickness-k	N	[60]	N	[GJ]	P	[T]	N	[10]	N	[GJ]	N	[49]	N	[GJ]	N	[GJ]	N	[7]	N	[GJ]	O?	
Genus-k	P	[34]	N	[GJ]	P	[T]	N	[10]	N	[GJ]	O?		N	[GJ]	N	[GJ]	O?		N	[GJ]	P	[61]
Perfect	O!		P	[42]	P	[42]	P	[42]	P	[42]	O?		N	[1]	N	[14]	O?		N	[GJ]	I	[GJ]
Chordal	P	[76]	P	[40]	P	[40]	P	[40]	P	[40]	O?		N	[22]	N	[14]	O?		N	[83]	I	[GJ]
Split	P	[40]	P	[40]	P	[40]	P	[40]	P	[40]	O?		N	[22]	N	[19]	O?		N	[83]	I	[15]
Strongly Chordal	P	[31]	P	[40]	P	[40]	P	[40]	P	[40]	O?		O?		P	[32]	O?		P	[83]	O?	
Comparability	P	[40]	P	[40]	P	[40]	P	[40]	P	[40]	O?		N	[1]	N	[28]	O?		N	[GJ]	I	[GJ]
Bipartite	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	N	[1]	N	[28]	P	[T]	N	[GJ]	I	[GJ]
Permutation	P	[40]	P	[40]	P	[40]	P	[40]	P	[40]	O?		O		P	[33]	O?		P	[23]	P	[21]
Cographs	P	[T]	P	[40]	P	[40]	P	[40]	P	[40]	O?		P	[25]	P	[33]	O?		P	[23]	P	[25]
Undirected Path	P	[39]	P	[40]	P	[40]	P	[40]	P	[40]	O?		O?		N	[16]	O?		O?		I	[GJ]
Directed Path	P	[38]	P	[40]	P	[40]	P	[40]	P	[40]	O?		O?		P	[16]	O?		P	[83]	O?	
Interval	P	[17]	P	[44]	P	[44]	P	[44]	P	[44]	O?		P	[53]	P	[16]	O?		P	[83]	P	[57]
Circular Arc	P	[78]	P	[44]	P	[50]	P	[44]	N	[36]	O?		O?		P	[13]	O?		P	[83]	O?	
Circle	P	[71]	P	[GJ]	P	[50]	O?		N	[36]	O?		P	[12]	O?		O?		P	[70]	O?	
Proper Circ. Arc	P	[77]	P	[44]	P	[50]	P	[44]	P	[66]	O?		P	[12]	P	[13]	O?		P	[83]	O?	
Edge (or Line)	P	[47]	P	[GJ]	P	[T]	N	[GJ]	N	[49]	O?		N	[11]	N	[GJ]	O?		N	[70]	I	[15]
Claw-Free	P	[T]	P	[63]	O?		N	[GJ]	N	[49]	O?		N	[11]	N	[GJ]	O?		N	[70]	I	[15]

Graph Class	Member		IndSet		Clique		CliPar		ChrNum		ChrInd		HamCir		DomSet		MaxCut		StTree		GraphIso	
Trees/Forests	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[GJ]	P	[T]	P	[GJ]
Almost Trees (k)	P		P	[OG]	P	[T]	P	[6]	P	[OG]	P	[7]	P	[OG]	P	[OG]	P	[8]	P	[9]	P	[7]
Partial k-trees	P	[OG]	P	[OG]	P	[T]	P	[6]	P	[OG]	P	[7]	P	[OG]	P	[OG]	P	[8]	P	[9]	P	[7]
Bandwidth-k	P	[OG]	P	[OG]	P	[T]	P	[6]	P	[OG]	P	[7]	P	[OG]	P	[OG]	P	[OG]	P	[9]	P	[OG]
Degree-k	P	[T]	N	[GJ]	P	[T]	N	[GJ]	N	[GJ]	N	[OG]	N	[GJ]	N	[GJ]	N	[GJ]	N	[10]	P	[OG]
Planar	P	[GJ]	N	[GJ]	P	[T]	N	[OG]	N	[GJ]	O		N	[GJ]	N	[GJ]	P	[GJ]	N	[10]	P	[GJ]
Series Parallel	P	[OG]	P	[OG]	P	[T]	P	[6]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[GJ]	P	[OG]	P	[GJ]
Outerplanar	P		P	[OG]	P	[T]	P	[OG]	P	[OG]	P	[OG]	P	[T]	P	[OG]	P	[GJ]	P	[OG]	P	[GJ]
Halin	P		P	[OG]	P	[T]	P	[OG]	P	[OG]	P	[OG]	P	[T]	P	[OG]	P	[GJ]	P	[11]	P	[GJ]
k-Outerplanar	P		P	[OG]	P	[T]	P	[OG]	P	[OG]	P	[7]	P	[OG]	P	[OG]	P	[GJ]	P	[9]	P	[GJ]
Grid	P		P	[GJ]	P	[T]	P	[T]	P	[T]	P	[GJ]	N	[OG]	N	[OG]	P	[T]	N	[10]	P	[GJ]
$K_{3,3}$-Free	P	[OG]	N	[GJ]	P	[T]	N	[GJ]	N	[GJ]	O?		N	[GJ]	N	[GJ]	P	[OG]	N	[10]	I	[12]
Thickness-k	N	[OG]	P	[GJ]	P	[T]	N	[GJ]	N	[GJ]	N	[OG]	N	[GJ]	N	[GJ]	N	[OG]	N	[10]	O?	
Genus-k	P	[OG]	P	[GJ]	P	[T]	N	[GJ]	N	[GJ]	O?		N	[GJ]	N	[GJ]	O?		N	[10]	P	[OG]
Perfect	P	[13]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	N	[14]	N	[OG]	N	[OG]	N	[8]	N	[10]	I	[GJ]
Chordal	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		N	[OG]	N	[OG]	N	[8]	N	[15]	I	[GJ]
Split	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		N	[OG]	N	[OG]	N	[8]	N	[15]	I	[OG]
Strongly Chordal	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		N	[16]	P	[OG]	N	[4]	P	[15]	I	[17]
Comparability	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	N	[14]	N	[OG]	N	[OG]	N	[18]	N	[10]	I	[GJ]
Bipartite	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	N	[OG]	N	[OG]	P	[T]	N	[10]	I	[GJ]
Permutation	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[19]	P	[OG]	O?		P	[20]	P	[OG]
Cographs	P	[T]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]	P	[8]	P	[20]	P	[OG]
Undirected Path	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		N	[21]	N	[OG]	N	[8]	N	Thm. ??	I	[GJ]
Directed Path	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		N	[22]	P	[OG]	O?		P	[15]	O?	
Interval	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]	O?		P	[15]	P	[OG]
Circular Arc	P	[OG]	P	[OG]	P	[OG]	P	[OG]	N	[OG]	O?		P	[23]	P	[OG]	O?		P	[15]	O?	
Circle	P	[OG]	P	[GJ]	P	[OG]	N	[24]	N	[OG]	O?		P	[OG]	N	[25]	N	[26]	P	[OG]	O?	
Proper Circ. Arc	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]	O?		P	[15]	P	[5]
Edge (or Line)	P	[OG]	P	[GJ]	P	[T]	N	[GJ]	N	[OG]	N	[14]	N	[OG]	N	[GJ]	P	[27]	N	[OG]	I	[OG]
Claw-Free	P	[T]	P	[OG]	N	[28]	N	[GJ]	N	[OG]	N	[14]	N	[OG]	N	[GJ]	N	[8]	N	[29]	I	[OG]

Dániel Marx plenary talk at ICGT 2014

Every graph is easy or hard:
dichotomy theorems for graph problems

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Dichotomy theorems

- Dichotomy theorems give good research programs: easy to formulate, but can be hard to complete.
- The search for dichotomy theorems may uncover algorithmic results that no one has thought of.
- Proving dichotomy theorems may require good command of both algorithmic and hardness proof techniques.

Graph Class	Member		IndSet		Clique		CliPar		ChrNum		ChrInd		LongPath		DomSet		MaxCut		StTree		GraphIso	
Trees/Forests	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[GJ]	P	[T]	P	[GJ]
Partial k-trees	P	[OG]	P	[OG]	P	[T]	**P**	[6]	P	[OG]	**P**	[7]	P	[OG]	P	[OG]	**P**	[8]	**P**	[9]	FPT	[32]
Almost Trees (k)	P		P	[OG]	P	[T]	**P**	[6]	**P**	[OG]	**P**	[7]	**P**	[OG]	P	[OG]	**P**	[8]	**P**	[9]	**P**	[7]
Bandwidth-k	P	[OG]	P	[OG]	P	[T]	**P**	[6]	P	[OG]	**P**	[7]	**P**	[OG]	P	[OG]	P	[OG]	**P**	[9]	P	[OG]
Series Parallel	P	[OG]	P	[OG]	P	[T]	**P**	[6]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[GJ]	P	[OG]	P	[GJ]
Outerplanar	P		P	[OG]	P	[T]	P	[OG]	P	[OG]	P	[OG]	P	[T]	P	[OG]	P	[GJ]	P	[OG]	P	[GJ]
Halin	P		P	[OG]	P	[T]	P	[OG]	P	[OG]	P	[OG]	P	[T]	P	[OG]	P	[GJ]	**P**	[11]	P	[GJ]
k-Outerplanar	P		P	[OG]	P	[T]	P	[OG]	P	[OG]	**P**	[7]	P	[OG]	P	[OG]	P	[GJ]	**P**	[9]	P	[GJ]
Planar	P	[GJ]	FPT	[33]	P	[T]	O*	[OG]	PNP	[34]	O		FPT	[35]	FPT	[36]	P	[GJ]	FPT	[37]	P	[GJ]
Grid	P		P	[GJ]	P	[T]	P	[T]	P	[T]	P	[GJ]	FPT	[35]	FPT	[36]	P	[T]	FPT	[37]	P	[GJ]
$K_{3,3}$-Free	P	[OG]	W[1]	[38]	P	[T]	O*	[GJ]	PNP	[34]	O?		FPT	[35]	FPT	[39]	P	[OG]	XP	[T]	FPT	[32]
Thickness-k	PNP	[OG]	P	[GJ]	P	[T]	O*	[GJ]	PNP	[34]	PNP	[OG]	FPT	[35]	FPT	[40]	FPT	[41]	FPT	[37]	FPT	[32]
Genus-k	P	[OG]	P	[GJ]	P	[T]	O*	[GJ]	PNP	[34]	O?		FPT	[35]	FPT	[40]	FPT	[41]	FPT	[37]	P	[OG]
Degree-k	P	[T]	FPT	[33]	P	[T]	O*	[GJ]	PNP	[34]	PNP	[42]	FPT	[35]	FPT	[43]	FPT	[41]	FPT	[44]	P	[OG]
Perfect	**P**	[13]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	**O∗**	[14]	FPT	[35]	W[2]	[45]	FPT	[41]	W[2]	[45]	FPT	[32]
Chordal	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		FPT	[35]	W[2]	[45]	FPT	[41]	W[2]	[45]	FPT	[32]
Split	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		FPT	[35]	W[2]	[45]	FPT	[41]	W[2]	[45]	FPT	[32]
Strongly Chordal	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		FPT	[35]	P	[OG]	FPT	[41]	P	[15]	FPT	[32]
Comparability	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	**O∗**	[14]	FPT	[35]	W[2]	[46]	FPT	[41]	XP	[T]	FPT	[32]
Bipartite	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	FPT	[35]	W[2]	[46]	P	[T]	XP	[T]	FPT	[32]
Permutation	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		**P**	[19]	P	[OG]	FPT	[41]	P	[20]	P	[OG]
Cographs	P	[T]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]	**P**	[8]	P	[20]	P	[OG]
Undirected Path	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		FPT	[35]	XP	[T]	FPT	[41]	XP	[T]	FPT	[32]
Directed Path	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		FPT	[35]	P	[OG]	FPT	[41]	P	[15]	FPT	[32]
Interval	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]	FPT	[41]	P	[15]	P	[OG]
Circular Arc	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O*	[OG]	O?		**P**	[23]	P	[OG]	FPT	[41]	P	[15]	FPT	[32]
Circle	P	[OG]	P	[GJ]	P	[OG]	XP	[24]	O*	[OG]	O?		P	[OG]	W[1]	[47]	FPT	[41]	P	[OG]	FPT	[32]
Proper Circ. Arc	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]	FPT	[41]	P	[15]	**P**	[5]
Edge (or Line)	P	[OG]	P	[GJ]	P	[T]	O*	[GJ]	PNP	[42]	O*	[14]	FPT	[35]	FPT	[48]	**P**	[27]	XP	[T]	FPT	[32]
Claw-Free	P	[T]	P	[OG]	FPT	[48]	PNP	[49]	PNP	[42]	O*	[14]	FPT	[35]	FPT	[48]	FPT	[41]	XP	[T]	FPT	[32]