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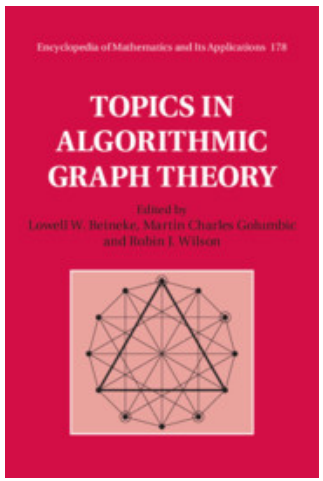
Topics in Algorithmic Graph Theory

# Chapter 3 · Total colouring

by Celina M. H. de Figueiredo

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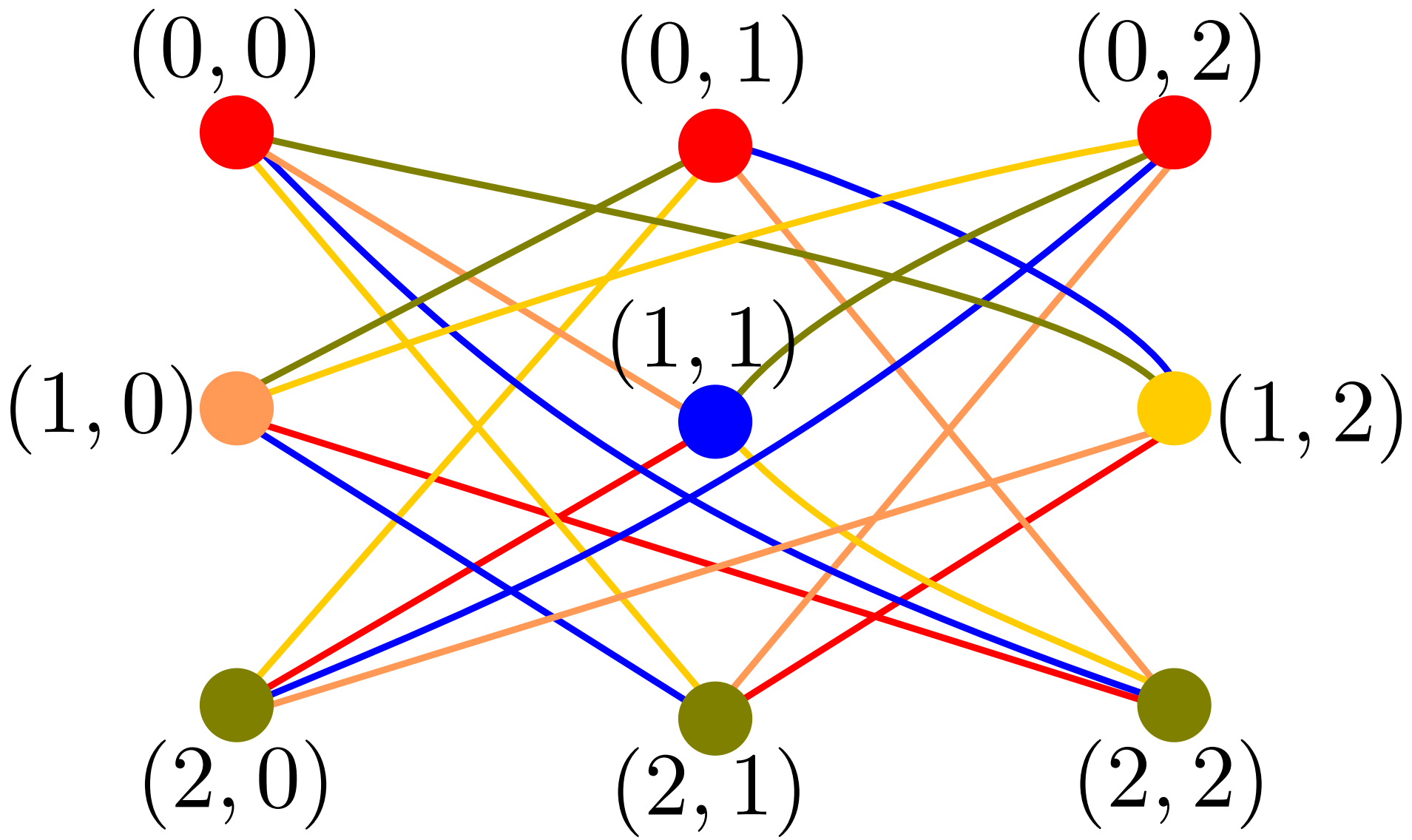



a 5-total coloring of $C_3 \times C_3$

# 2 · Hilton's condition

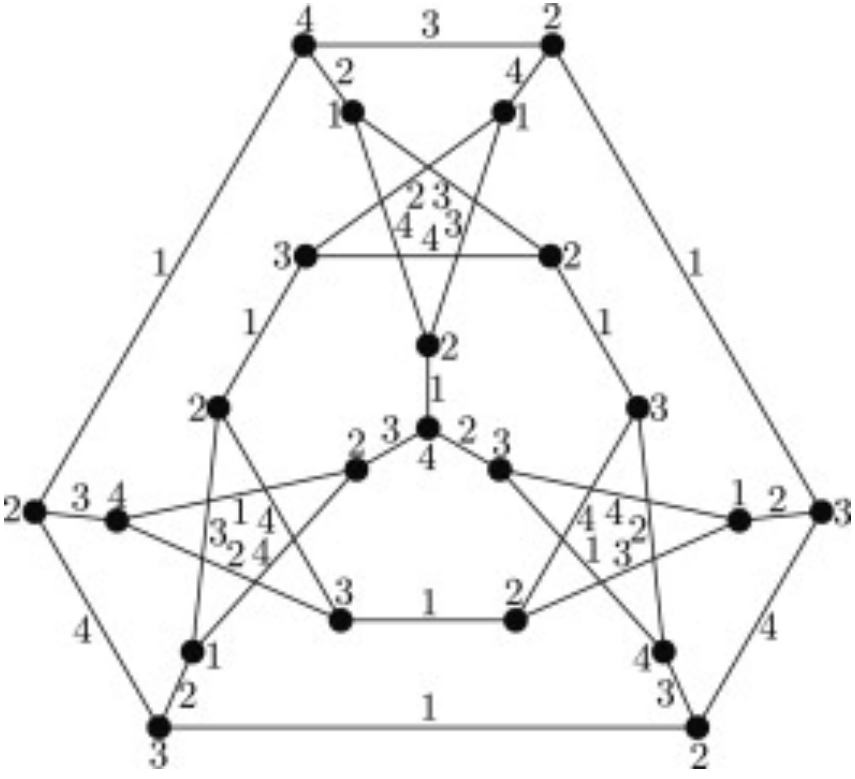
A graph satisfies Hilton's condition if the subgraph induced by the closed neighbourhood of a vertex of maximum degree is of type 2.

| Graph class | even $\Delta$ | odd $\Delta$ |
|---|---|---|
| complete | type 1 | type 2 (Hilton's condition) |
| universal vertex | type 1 | Hilton's condition [1] |
| split | type 1 | open |
| indifference | type 1 | open |
| split-indifference | type 1 | Hilton's condition [2] |
| 3-clique graph | type 1 | open |

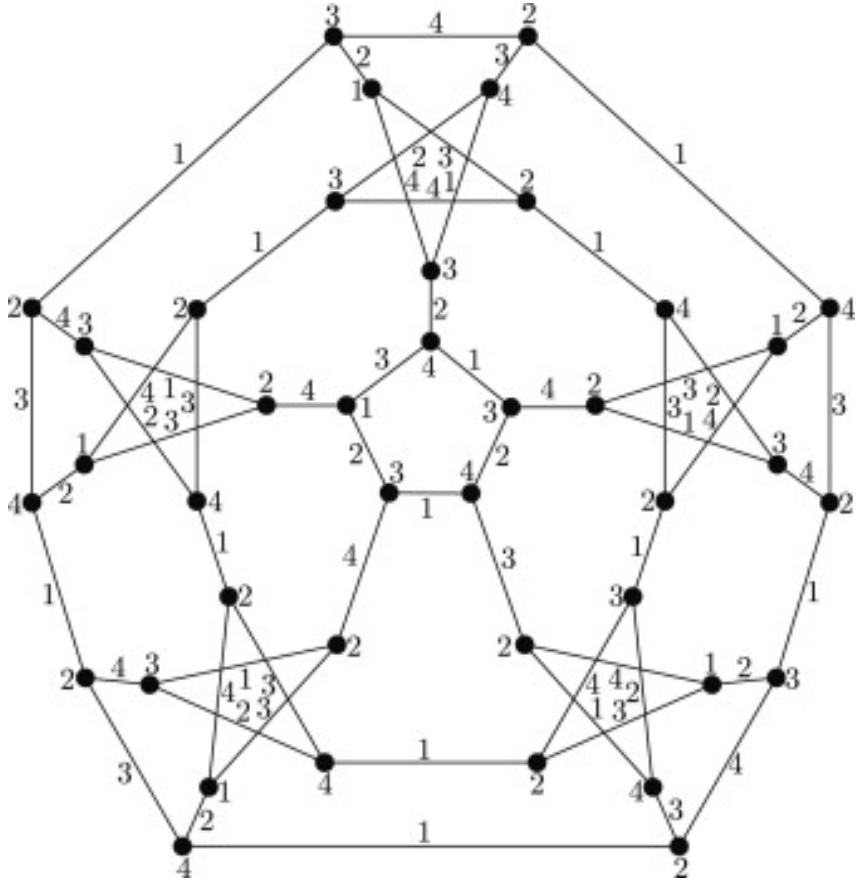
classes with respect to Hilton's condition on total colouring

[1] A. J. W. Hilton, A total-chromatic number analogue of Plantholt's theorem, Discrete Math. 79 (1990)
[2] C. N. Campos, C. H. de Figueiredo, R. Machado and C. P. Mello, The total chromatic number of split-indifference graphs, Discrete Math. 312 (2012)

# 3 · Cubic graphs



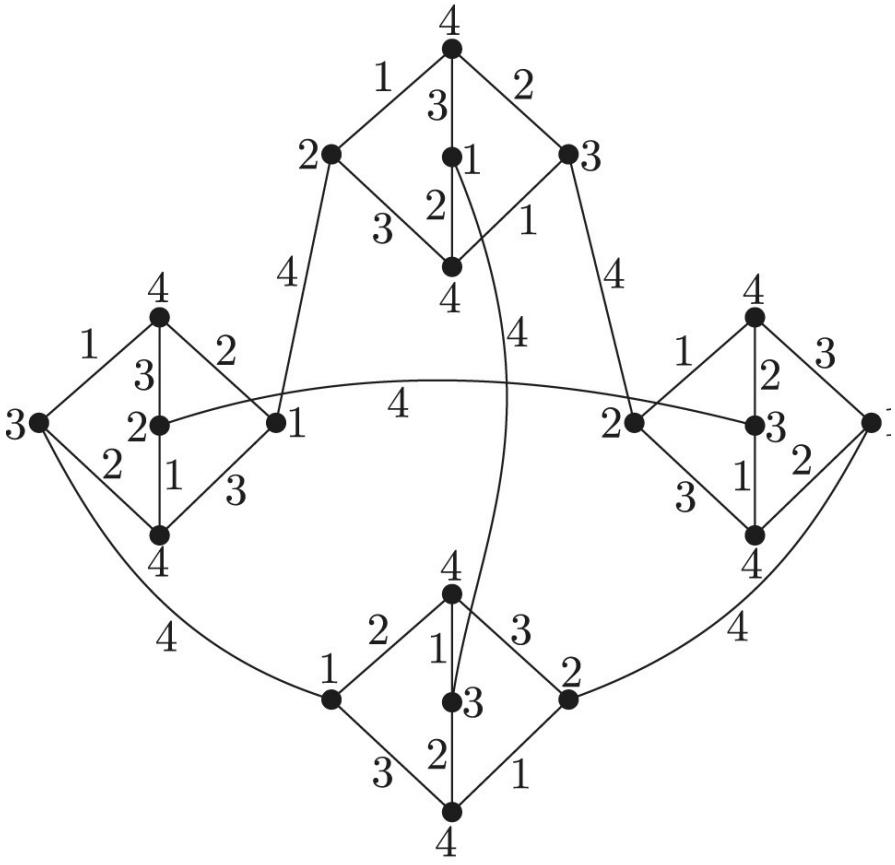
a 4-total-coloring of a smallest Loupekhine snark



a 4-total-coloring of a smallest Goldberg snark

# 4 · Equitable total colourings

Question 4.1 Is there a cubic graph of type 1 with girth greater than 4 and equitable total chromatic number 5?



type 1 cubic graph of equitable total chromatic number 5

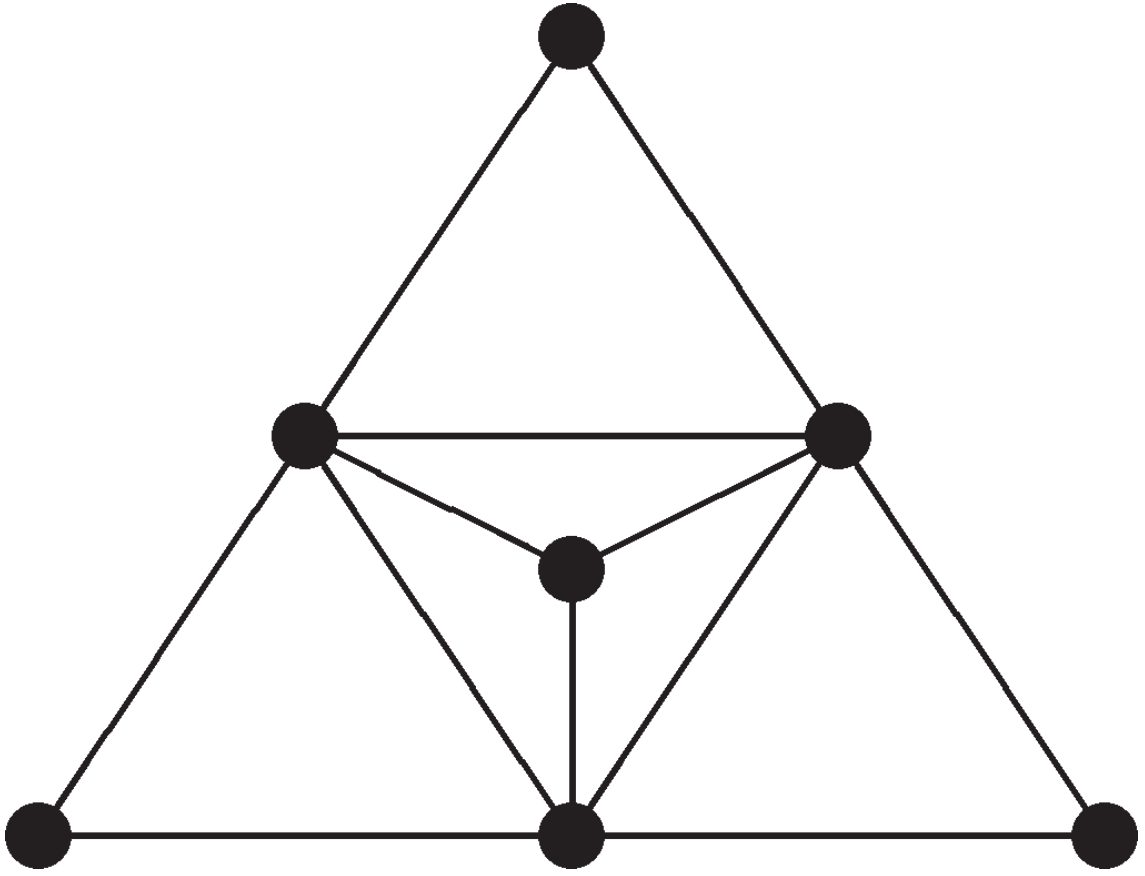
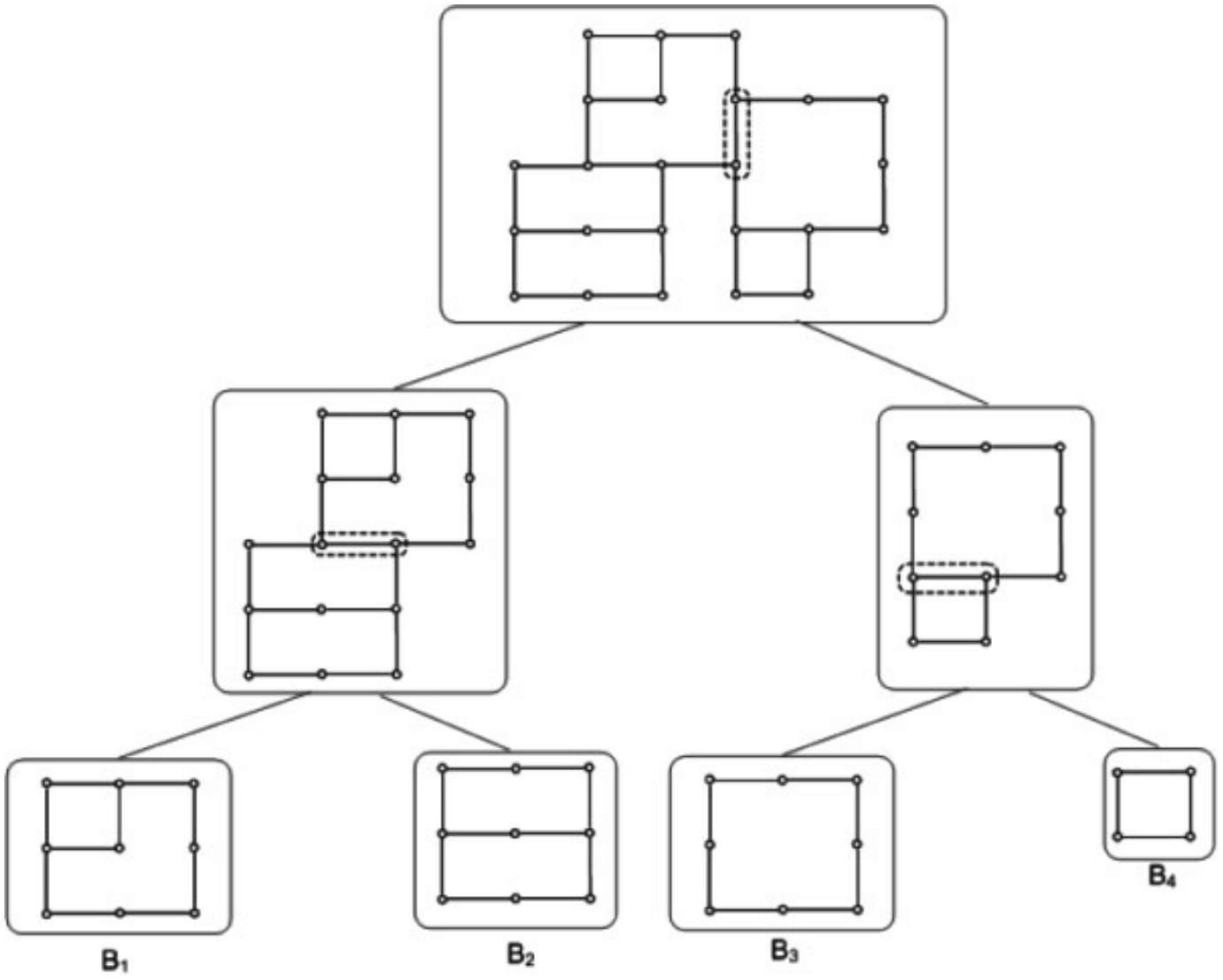


Fig. 1. A chordal graph with $\chi(G^2) > \Delta(G) + 1$.

# 6 · Decomposition




a decomposition tree with respect to clique 2-cutsets

# 7 · Complexity separation

| class \ problem | edge-colouring | total colouring |
|---|---|---|
| unichord-free | NP-complete [1] | NP-complete [2] |
| chordless | polynomial [3] | polynomial [3] |
| {square,unichord}-free | NP-complete [1] | polynomial [4] |
| bipartite unichord-free | polynomial | NP-complete [5] |

the computational complexity of colouring problems
restricted to subclasses of unichord-free graphs


[1] R. C. S. Machado, C. M. H. de Figueiredo and K. Vušković, Chromatic index of graphs with no cycle with unique chord, Theoret. Comput. Sci. 411 (2010)
[2] R. C. S. Machado and C. M. H. de Figueiredo, Total chromatic number of unichord-free graphs, Discrete Appl. Math. 159 (2011)
[3] R. C. S. Machado, C. M. H. de Figueiredo and N. Trotignon, Edge-colouring and total-colouring chordless graphs, Discrete Math. 313 (2013)
[4] R. C. S. Machado, C. M. H. de Figueiredo and N. Trotignon, Complexity of colouring problems restricted to unichord-free and {square,unichord}-free graphs, Discrete Appl. Math. 164 (2014)
[5] R. C. S. Machado and C. M. H. de Figueiredo, Complexity separating classes for edge-colouring and total-colouring, J. Brazil. Comp. Soc. 17 (2011)

# 8 · Concluding remarks and conjectures

Question 8.1 Are all partial grids with maximum degree 3 of type 1?

Question 8.2 Are all chordal graphs with even maximum degree of type 1?

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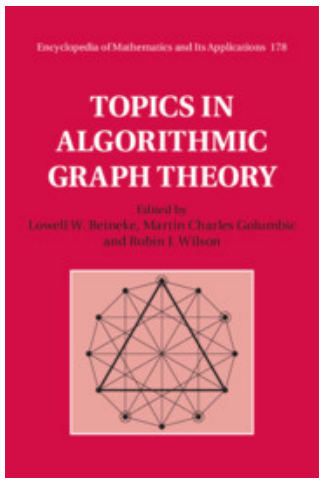


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